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Superdirective Arrays: The Use of Decoupling between Elements to Ease Design and Increase Bandwidth

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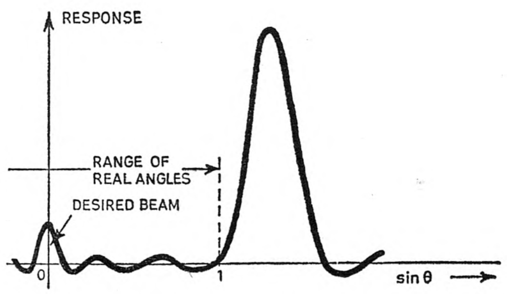
Summary: The design and achievement of superdirective receiving arrays for radio or acoustic waves can be greatly eased by the use of decoupling arrangements between the elements, such as buffer amplifiers in the output of each element before the outputs are combined to form the directional pattern. By this means, too, the severe bandwidth restrictions normally associated with superdirectivity can be removed.

1. Introduction

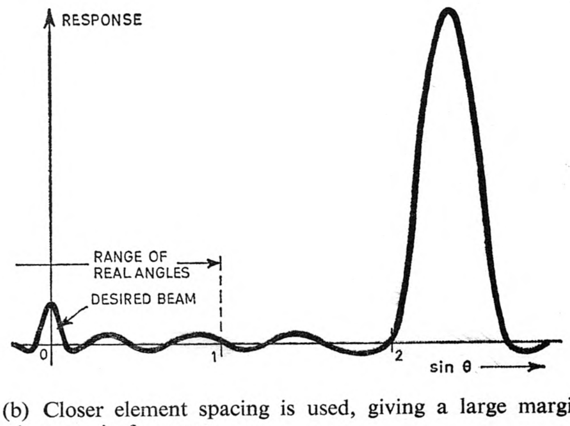
The concept of superdirectivity is quite old, discussions of it occurring in numerous text-books on radio antennas,[1] and the literature of the subject is vast.[2] Nevertheless little practical use has been made of superdirective arrays either in radio or in acoustics, and experimental results are conspicuously sparse in the literature. Indeed, no very large degree of superdirectivity appears ever to have been achieved. Some of the practical difficulties of achieving a large superdirective gain are fairly easy to see, but some are rather obscure. Even when obtained, superdirectivity is usually assumed to be of little value because of the very narrow bandwidths apparently inherent in it. This note is intended to show that some of the difficulties normally associated with superdirectivity can be eliminated (or at any rate largely eliminated) by the use of decoupling (or buffer amplifiers) between the elements of the array.

A formal and precise definition of superdirectivity is difficult to give, but there seems no doubt that what is usually meant by the term is the property of obtaining a narrower beam (or higher directivity factor) from a linear array than is obtained with uniform or smoothly-tapered excitation, by crowding zeros (or null responses) into its directional pattern in the range of real angles at the expense of very large responses outside the range of real angles. Naturally, such responses outside the range of real angles cannot be directly observed, but what is meant becomes clear if the response of the array is plotted as a function, not of θ, the direction in space, but of $\sin\theta$. This is done in Fig. 1 for a broadside array, where $\theta = 0$ in the broadside direction. The directional pattern which can be measured is evidently that corresponding to angles for which $|\sin\theta| \leq 1$, so that $\sin\theta = \pm 1$ defines the limits of the range of real angles. For values of θ where $|\sin\theta| > 1$, i.e. where θ is not a real angle, the response can still be calculated although it cannot be directly measured.

Methods of calculating the required excitation (for transmitting) or sensitivity (for receiving) of the various elements of the array in order to obtain a desirable directional pattern are adequately described in the literature[3–5] and need not be discussed here. One feature which is fundamental to superdirectivity needs to be mentioned however. This is that the element spacing must be less than one half-wavelength and that alternate elements (or occasionally alternate groups of elements) must be connected in opposite phase, or at any rate with an antiphase component. It is not difficult to see that this is associated with the



(a) Major response in the range $|\sin\theta| > 1$ but as near as possible to the range of real angles.



(b) Closer element spacing is used, giving a large margin for increase in frequency.

Fig. 1. Superdirective directional responses plotted against $\sin\theta$, where θ is the angle of a particular wave direction relative to the broadside direction.

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placing of large responses in the range of angles for which $|\sin \theta| > 1$. It is also easy to see that if alternate elements are in phase opposition, then whatever magnitudes of response are given to the individual elements (and these are usually unequal), the resultant far-field peak response in the broadside direction must be considerably less that it would be for the same elements used with the same currents in the more normal co-phasal manner. This leads to some important practical objections to superdirective arrays, as follows:

(i) Since for given currents in the elements the losses are fixed, a superdirective transmitting array gives a smaller peak transmission for the same losses as a normal array. It is therefore less efficient.

(ii) Since for given signal levels on the individual elements, the resultant output of a superdirective receiving array is smaller than for a normal array, its noise figure is worse. (Noise figure is a measure of the performance of an array in respect to thermal noise generated in the loss resistances of its elements and has the same general significance as the noise figure of an amplifier.)

(iii) Since the peak response is formed by differences between elements, and not by the additions as in a normal array, the response is very sensitive to small errors or variations in the individual elements or their associated circuits.

There may be circumstances where these objections are unimportant. For example, in a sonar system operating at low acoustic frequencies, say below 50 kHz, the acoustic noise level in the water due to wave action, reflected ship's noise, etc., usually so greatly exceeds any thermal noise in the receiver that a very considerable worsening of the noise figure of the array is quite acceptable. Special circumstances, or even careful design in ordinary circumstances, could reduce the importance of objection (iii).

There are other difficulties normally associated with superdirectivity which are usually regarded as fundamental. These are:

(a) the effect of the mutual coupling between elements in preventing the desired operation of the array from being achieved, and

(b) the very narrow bandwidth of a superdirective array as compared with the same array and elements used normally.

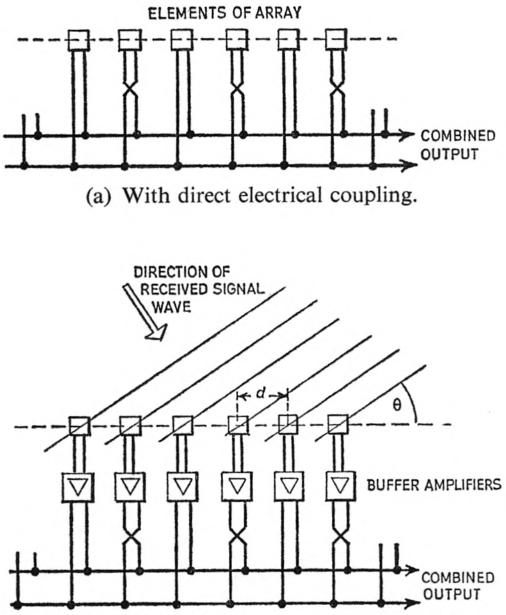
It must be obvious that the mutual coupling of elements spaced at fractions of a wavelength is considerable. In a normal array, where the elements are co-phasal, this can have a serious enough effect on the operation of the array,[6,7] but with superdirective operation the effect is clearly very great. This makes the design and setting-up of a superdirective array very difficult, although with care and in simple cases it can be successfully achieved.[8]

The narrow bandwidth associated with superdirectivity is perhaps harder to understand. It arises because the radiation resistance is very low and the reactance large, thus making tuning-out of the reactance necessary; the system then becomes a high-Q resonant system with consequently narrow bandwidth. We need, therefore, first to see why the radiation resistance is low. It is easier to see this for a transmitting array, and then invoke the principle of reciprocity to justify the same low resistance for the same array used for reception. On transmission we have alternate elements caused to carry currents (if they are radio antennas) or to vibrate (if they are electro-acoustic transducers) in opposite phase; yet they are very close together and thus have large mutual coupling. This means that even in the vicinity of one element the resultant field intensity or acoustic pressure is partially cancelled and shifted in phase, so that its in-phase component is small in relation to the current in the element (or to its velocity if it is acoustic). This explains the low radiation resistance, and also suggests a considerable radiation reactance. Most authors (e.g. Woodward and Lawson[4]) refer to the large reactive field in the vicinity of the aperture; Schelkunoff and Friis[1] describe it as resonant, but Woodward and Lawson state that 'a large balancing reactance would have to be incorporated behind the aperture plane, thus forming a highly-resonant and therefore frequency-sensitive arrangement'.

2. The Method of Overcoming the Limitations of Mutual Coupling and Narrow Bandwidth in Superdirective Arrays

It seems that neither the mutual coupling effect nor the narrow-bandwidth effect discussed above are really fundamental in the design and operation of superdirective receiving arrays, since both can be kept exactly the same as in the corresponding non-superdirective array merely by interposing buffer amplifiers (or any other suitable decoupling arrangements) between the elements and their electrical interconnections as shown in Fig. 2(b). The taper function and its phase reversals can be inserted without affecting the mutual couplings and radiation impedances at all. The bandwidth of the superdirective array is then exactly the same as that of the same array used without superdirectivity.† The large reactive field in the vicinity of the array does not now exist, since the currents or vibrations in the elements

† This is not to say, of course, that non-superdirective arrays with very close element spacings of only a fraction of a wavelength do not have complicated performance phenomena. Rusby[6] gives an interesting example, where mutual couplings in a closely-spaced rectangular array cause one element to have a negative radiation resistance.



(a) With direct electrical coupling.

(b) With buffer amplifiers.

Fig. 2. Superdirective array connections.

are now co-phasal. Yet the directional response of the array is undoubtedly the same as that of the superdirective array of the conventional type (if the latter could be successfully constructed).

The writer cannot see any rigorously corresponding system or reasoning for a transmitting array and it is possible that reciprocity fails here. But an approach to a reciprocal system can be obtained by using a separate transmitting amplifier for each element, with each amplifier output approximating to a constant-current drive, so that each element has an amplitude largely independent of the oscillation of other elements.

It is interesting to note that another system for overcoming the effects of mutual coupling (and incidentally also the reactive field) in receiving arrays has been proposed[9] and tried experimentally.[10] This system is not of general application, however, as it uses different frequencies for the signals on the different elements, and thus requires the 'co-operation' of the sending end.

3. Analysis

A simple analysis of the system of Fig. 2(b) is as follows: Assume that the number (r) of elements in the array is fairly large, and that the complex coupling coefficient (expressed as an equivalent voltage-transfer ratio) is m_1 between adjacent elements, m_2 between elements spaced by two inter-element distances, m_3 by three, and so on. Let v_n be the voltage output of the nth element when the incoming wave is not allowed to excite any other element, and v'_n the voltage when all elements are excited. Then when all elements receive the wave, their outputs are each given by

$$v'_n = v_n + m_1(v'_{n-1} + v'_{n+1}) + m_2(v'_{n-2} + v'_{n+2}) + \\ + m_3(v'_{n-3} + v'_{n+3}) + \ldots\ldots \quad (1)$$

where the terms continue until the subscripts of the v' terms reach 1 or r. It is easiest to assume at this stage that all elements have equal sensitivity and that the taper function (T_n for element n) is imposed after the buffer amplifiers.

Now a rigorous solution of the set of n equations represented by eqn. (1) is evidently very complicated, and it is quite unnecessary for our present argument. We assume that the signal is a coherent plane wave coming from a direction making an angle θ with the normal to the array, and we put

$$\phi = (2\pi d/\lambda)\sin\theta \quad \ldots\ldots(2)$$

where d is the spacing between the centres of the elements and λ is the wavelength. Let the magnitude of v_n resulting from this signal be v_S for every element. Then just to see what form the solution of eqn. (1) should take, assume for a moment that the modulus of v'_n is the same for all values of n, and that the phase interval ϕ applies equally to adjacent values of v' as to adjacent values of v. Then as a first approach to the solution we may derive from eqn. (1) this relationship:

$$v'_n = v_S \exp(jn\phi)/ \\ [1 - 2m_1\cos\phi - 2m_2\cos 2\phi - 2m_3\cos 3\phi - \ldots] \ldots (3)$$

where the series in m_1, m_2, etc. continues only according to the relation of n to the end elements 1 and r; residual terms in the series are of the form $m_k \exp(\pm jk\theta)$. We see, therefore, that the true solution is of the form:

$$v'_n = v_S \exp(jn\phi) \,.\, A(n, m, \phi) \quad \ldots\ldots(4)$$

where $A(n,m,\phi)$ is a function of the m-values and of ϕ which is different for each value of n.

The total output after combining the signals from the buffer amplifiers, with the polarity reversals as indicated in Fig. 2(b), is proportional to

$$v_S \sum_{n=1}^{r} A(n, m, \phi) \,.\, (-1)^n \,.\, T_n \exp(jn\phi) \quad \ldots\ldots(5)$$

Now if the number of elements is infinite, A is independent of n and is

$$A(m, \phi) = 1/[1 - 2\sum_{k=1}^{\infty} m_k \cos k\phi] \quad \ldots\ldots(6)$$

In other words, the asymmetry of the mutual coupling towards the ends of the array is ignored when the array is infinite. Since in practice the values m_1, m_2, ... m_k, ... can be expected to diminish fairly rapidly as k increases, eqn. (6) is probably an adequate approximate representation for arrays with as few as six or seven elements. We can then write the total output as

$$v_S \, . \, A(m, \phi) \, . \, D(\phi) \qquad \ldots\ldots(7)$$

where $D(\phi)$ is the superdirective directional function,

$$D(\phi) = \sum_{n=1}^{r} (-1)^n \, . \, T_n \exp(jn\phi) \qquad \ldots\ldots(8)$$

It should be noted that, since $d < \lambda/2$ in a superdirective array, the term $A(m, \phi)$ may well have little effect on the overall directivity of the array; we would normally expect $D(\phi)$ to be the dominant directional response.

It is clear that $D(\phi)$ is the only part of the response shown in eqn. (7) which is affected by making the array superdirective, and there is no basic frequency limitation in $D(\phi)$. It is, of course, necessary that, over the frequency range in which operation is intended, the large responses (shown in Fig. 1) which occur for $|\sin \theta| > 1$ should not inadvertently enter the range of real angles. This can be avoided by suitable design as shown in exaggerated form in (b) in Fig. 1, where a large margin for increase in frequency has been allowed. The condition of (a) in Fig. 1 should occur only at the lowest frequency.

It is probable that the array using buffer amplifiers, while enabling the desired superdirective directional pattern to be obtained, actually gives a lower gain. Here 'gain' is distinguished from 'directional gain' or directivity as being the relation between output from the array and the strength of the incident field, for normal incidence. For the conventional superdirective array, but with loss-free elements, Schelkunoff and Friis state that the resonance effect and low radiation resistance 'enable the antenna to create a strong reactive field extending to large distances from the antenna which re-directs the power passing through a large area of the incoming plane wave and forces it to flow toward the antenna. Detuned superdirective antennas intercept but little power'. This explains how, in a loss-free array, superdirectivity leads to an actual gain corresponding to the directional gain (or, in acoustic terms, to the increase in directivity index) over the same array used co-phasally. Thus the terms 'superdirectivity' and 'supergain' have tended to become synonymous. Under practical conditions the reactive field will not be so strong and effective, since the resistance losses in the elements are arithmetically additive and will tend to swamp the other effects. This is especially true of acoustic arrays, where efficiencies are usually much lower than in radio arrays. Experimental results are available for end-fire arrays, but the author knows of none for broadside arrays. In a radio end-fire array operating at 75 MHz, an actual gain (relative to a non-superdirective arrangement) of over 4 dB has been reported by Bloch, Medhurst and Pool[2] for a four-element array.

There is no doubt that the decoupled superdirective array cannot give an actual gain as compared with the same array used co-phasally, and indeed must give a

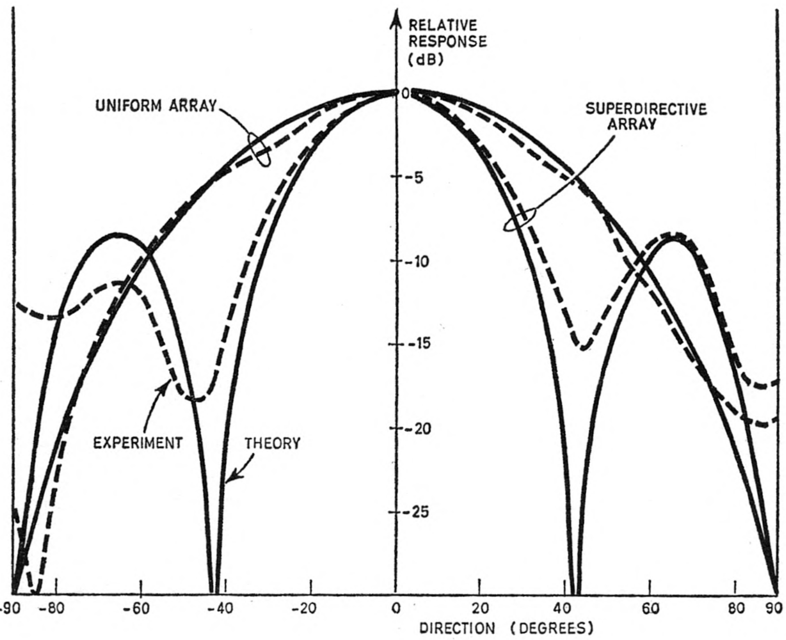


Fig. 3. Comparison of experimental and theoretical results for a 420 MHz array of three radio dipoles spaced at quarter-wave intervals. Buffer amplifiers were used as in Fig. 2(b). The superdirective array merely had the phase of the output from the central dipole reversed before adding to the others.

loss and cannot therefore be called a supergain array, even though its directivity index is the same as that of the normal superdirective arrangement. It is open to argument whether the decoupled array gives less or more gain than the normal superdirective array used without the reactive field effect being tuned out, thus also having a less restricted bandwidth. However, the use of decoupling certainly appears to simplify the design problem. The noise figure of the decoupled superdirective array may be somewhat worse than that of the normal superdirective array, but it is unlikely that the increase will be very significant in practice.

4. Experimental Results

No full experimental investigation of the proposals discussed above has yet been made and no experimental verification of the wider bandwidth which is predicted for the superdirective array is yet available. Some preliminary experiments have, however, been made by Mr. M. S. Pollard,[11] an undergraduate student in the author's department, and the results show that, at any rate, a simple superdirective array using buffering arrangements as in Fig. 2(b) does give the predicted directional pattern. The experimental arrangement comprised the central three half-wave radio dipoles in an array of seven (the others being terminated in dummy loads), spaced at one quarter-wavelength intervals, and operating at 420 MHz. The array was backed by a ground plane. The dipoles were connected via baluns, buffer amplifiers and hybrid arrangements to correspond in effect exactly to Fig. 2(b); the phase reversals could be removed when desired by suitable connections to the hybrids.

The three active elements had equal gains in the amplifier and hybrid systems, and if the outputs were connected *in phase*, a uniform normal array should be obtained. Allowing for the ground plane, the directional pattern should theoretically be as shown in the appropriate full-line curve in Fig. 3. The experimentally-measured pattern shown in the dashed curve agrees very well.

If the phase of the output of the central element is reversed, a simple superdirective system should be obtained which theoretically (allowing again for the ground plane) should have the directional response shown in Fig. 3 by the other full-line curve. This pattern is very much narrower in the main beam than that of the uniform array, but has bad secondary responses that would probably render it unattractive in practice. Nevertheless it makes a suitable basis for a test of the principle. The experimental pattern is shown in the appropriate dashed curve, and can be seen to be in reasonable agreement with the theoretical curve except that it has no real nulls. Thus there is no doubt that a superdirective response can be obtained this way and the discrepancies between theoretical and experimental patterns are not appreciably worse than in the case of the uniform array in spite of the very large mutual couplings. (The mutual impedances appeared, from indirect measurements, to be of the order of one-half the self-impedances.)

5. Conclusions

It has been shown that the design and achievement of superdirective receiving arrays can be greatly eased by the use of decoupling arrangements between the elements, and that by this means the severe bandwidth restrictions normally associated with superdirectivity can be removed. It is believed that the arguments are equally valid for radio or acoustic arrays. A distinction between superdirectivity and supergain has been made, and the decoupled array, while having the former, cannot have the latter.

6. Acknowledgments

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7. References

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