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Design of Simple Frequency-division-multiplex Communication Systems without Band-pass Filters, with particular reference to the Use of Constant-resistance Modulators

By

Professor D. G. TUCKER,
D.Sc., C.Eng., F.I.E.E., M.I.E.R.E.†

Summary: Consideration is given to the design problems of low-cost f.d.m. systems which achieve economy by the avoidance of band-pass filters for each channel at both ends of the system. It is shown that the best basis of design is the use of constant-resistance modulators.

1. Introduction

There is at the present time a considerable need for an economical multiplex transmission system for speech channels on junction circuits in the telephone network. The use of what were originally audio cables for wideband multi-channel systems involves a large number of problems, such as noise and inter-pair crosstalk, which become dominant when the frequencies involved are megahertz in place of the kilohertz for which the cable was designed. Because of these problems, the use of pulse-code modulation (p.c.m.) has been proposed and developed for such applications.[2] But this system sacrifices bandwidth for its relative insensitivity to noise and crosstalk, and typically requires a bandwidth of over 30 kHz multiplied by the number of channels. It is possible therefore that a cheap kind of frequency-division-multiplex (f.d.m.) system, possibly using double-sideband transmission but even so requiring only 8 kHz per channel, could be competitive in such an application. Separate 'go' and 'return' cables might be needed, and possibly more frequent repeaters, but the economics might still be in its favour.

There are doubtless other applications for a cheap f.d.m. terminal equipment, and not only for speech but also for telegraph and data signals.

In a normal f.d.m. terminal equipment, whether of the common single-sideband type or of the double-sideband type used in rural systems,[2] the channels are combined at the sending end and separated at the receiving end through band-pass filters. These filters are required to give a high performance and are normally of the type using crystal elements. They are expensive, and indeed may represent a very sizeable fraction of the total cost of the equipment.

The aim of the f.d.m. system design to be outlined in this paper is to avoid the need for channel band-pass filters and thus to permit the cost of the terminal equipment to be greatly reduced.

† Department of Electronic and Electrical Engineering, University of Birmingham.

2. General Requirements

As stated above, the band-pass filters normally used to separate channels in a frequency-division-multiplex carrier telephone system are expensive, and a system which can dispense with them without introducing other expensive items is therefore economically attractive. For single-sideband systems, band-pass filters can be dispensed with at the expense of introducing phase-shifting networks which are rather difficult to design but not necessarily expensive to make. For double-sideband systems band-pass filters can in principle be dispensed with without requiring other equipment in compensation. In both cases, however, successful operation without channel-separating filters is dependent on the use of low-pass filters (which are usually quite inexpensive) to limit the audio bandwidth to that appropriate to the channel spacing, on the avoidance of second-order modulation at the channel multiplexing terminals, and on the avoidance of direct rectification of channel signals with carrier leak; otherwise interchannel crosstalk occurs. If the overall frequency band of the group of channels at the multiplexing terminals is appreciably less than an octave, no crosstalk due to orders of modulation other than the second can occur.

One practical problem of some difficulty, therefore, is the avoidance of second-order modulation at the multiplexing terminals. If the carrier frequency of a particular channel is ω_p, and if ω_a represents an audio frequency, then two kinds of second-order modulation can exist:

(i) Second-order modulation of the audio signal, giving $2\omega_p \pm \omega_a$ at the multiplexing terminals; this does not cause interchannel crosstalk if the multiplex bandwidth is less than an octave by at least one channel bandwidth; it is in any case of a low level in a ring modulator being due either to second-harmonic components in the carrier supply or to unbalance in the modulator.

(ii) Second-order modulation of signals in the multiplexed band from other channels, due to

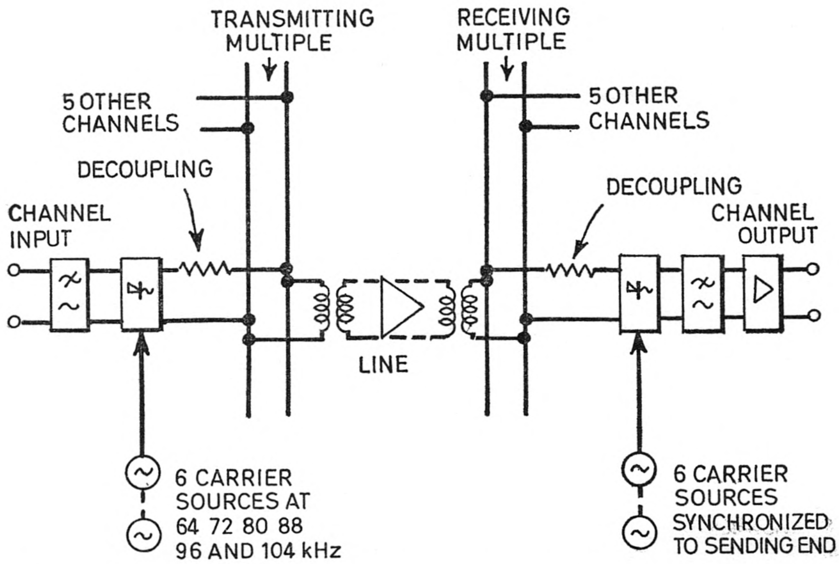


Fig. 1. General schematic of f.d.m. six-channel communication system without band-pass filters. The decoupling arrangement shown is only symbolic.

the variation with time of the modulator impedance seen at the multiplexing terminals. This applies equally to the sending and receiving ends of the system, and does cause inter-channel crosstalk. For example, the signal from channel n is of frequency $\omega_{pn} \pm \omega_a$; if this gets modulated by twice the carrier frequency of channel $(n+1)$, i.e. by $2(\omega_{pn}+\omega_s)$ where ω_s is the channel separation, then frequency $\omega_{pn}+2\omega_s \pm \omega_a$ is produced—which falls in channel $(n+2)$ and thus represents interchannel crosstalk.

It is clear, therefore, that it is the second kind of second-order modulation which limits the use of systems without band-pass filters in practice. If an ordinary channel modulator is used, with a low-pass filter of Zobel image type directly connected to the audio terminals of the modulator, the second-order modulation effect type (ii), when the other end of the modulator is nominally matched, produces interfering sidebands only about 12 dB below the main ones (see Appendix 2). For the crosstalk to be acceptable (of the order of 60 dB), it is necessary either

(*a*) to use modulators which produce very little second-order modulation at their input terminals, or

(*b*) to use decoupling resistors, attenuators, or buffer amplifiers in each channel.

The constant-resistance modulator, as described in some detail in other papers,[3,4] meets requirement (*a*) and its application to the present problem is dealt with in the following section.

A general schematic diagram of a six-channel system using double-sideband transmission and no band-pass filters is shown in Fig. 1. It occupies the frequency band from 60 to 108 kHz (which is the band used in the experimental work described later), but any band of less than an octave could be used.

3. F.D.M. System using Constant-resistance Ring Modulators

The use of constant-resistance ring modulators leads to a system design of maximum simplicity. This is because of the property that the impedance at one port of the modulator is a resistance which does not vary over the carrier cycle and so produces no modulation at this port. If, therefore, this port is connected directly to the channel-combining multiple at transmitting or receiving end, no interference with other channels is produced. In practice, of course, a perfectly constant resistance is not obtained, but it has been shown[4] that if the other port of each modulator is correctly terminated in a pure resistance, if suitable rectifiers are used, and signal levels are not excessive, then modulation of the input resistance of little more than 0·1% at the second harmonic of the carrier frequency (i.e. at $2\omega_p$) can be achieved. Such a low figure cannot be expected to be maintainable over a range of operating temperature and with commercial rectifiers used without special matching, but a figure of 1% ought to be maintainable.

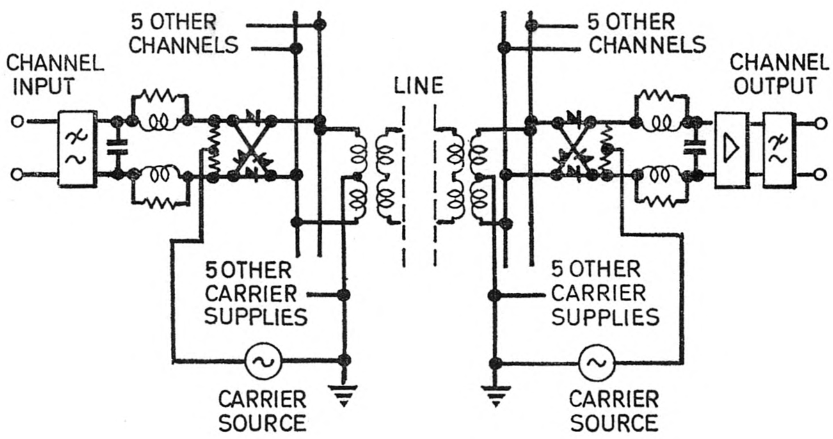


Fig. 2.

Circuit arrangement of six-channel system using constant-resistance modulators.

Note. Carrier supplies at the two ends must be synchronized in frequency and phase and must be free of interference from one to another to the extent demanded by the inter-channel crosstalk specification.

3.1. *Design Considerations*

The modulation of the input resistance of the modulator mentioned above is not a direct measure of the inter-channel crosstalk which it causes. Consider the receiving end of the system. The modulator input is fed from a low-resistance source (i.e. the multiple) comprising the other modulators in parallel with one another and with the output impedance of the line. Let this form a source resistance R_o and let the 'constant' resistance of the modulator be R_i. Then the actual crosstalk ratio, i.e. the ratio of voltage at a frequency $2\omega_p - \omega_q$, where ω_q is a sideband frequency, to voltage at the sideband frequency ω_q, is the modulation of the modulator resistance (at $2\omega_p$) reduced by the ratio $R_o/(R_o + R_i)$. In the six-channel system, assuming the line is matched to $R_i/6$, this ratio is 1/12. Expressed in decibels, the crosstalk should be 21·6 dB better than the modulation of R_i. Thus a 1% modulation (−40 dB) should lead to interchannel crosstalk, measured in terms of individual sidebands at the multiple, of about −61 dB. If the carrier frequencies are commensurate (i.e. all harmonics of a common basic frequency) the individual crosstalk sidebands add coherently in the channel in which they fall just as the wanted signal sidebands do, so that the overall crosstalk ratio due to the receiving end is also −61 dB. A similar amount of crosstalk would, however, be expected to be added at the transmitting multiple and, as this adds coherently, the overall system crosstalk would be about −55 dB.

It has been shown[4] that a suitable modulator can be made by using two rectifiers type OA85 in parallel as each element of a ring modulator, with a signal circuit impedance of around 18 kΩ, with carrier voltages across the rectifiers of the order of 100 mV and with signal levels of around 10 to 20 mV per channel. The carrier voltage can be fed via the centre point of the twin-wound 'office' winding of the line transformer; all six carrier supplies can be commoned at this point. At the other end of each modulator the use of a transformer should be avoided[3] in order to give the modulator as non-reactive a termination as possible, and a double resistance feed should be used. The arrangements are shown in detail in Fig. 2.

The arrangement of the low-pass filters and channel amplifiers in the individual channels needs some discussion. Since a non-reactive constant-resistance termination is required by the modulators on the audio side, the main filter should not be immediately adjacent to the modulator. At the receiving end the channel amplifier (which needs about 50 dB gain to give an output at zero dBm) could in principle follow the modulator directly, but in case it might be overloaded by the signals from other channels, or its input impedance should fall off and become reactive at the upper sideband of the output, $\omega_p + \omega_q$ which is of the order of $2\omega_p$, it may be advisable to precede it by a low-pass constant-resistance network[5] as shown. This network has a constant-resistance at all frequencies at the terminals facing the modulator, but can absorb stray (and other) capacitance at its other port. It has only a slow cut-off and will not be effective in restricting the output frequency range to below 4 kHz. To prevent signals from the adjacent channels appearing in the output as signals of frequency above 4 kHz, a low-pass filter should follow the amplifier or be built into it. Since these higher frequencies represent an unintelligible crosstalk at frequencies where the ear is less sensitive, a psopho-

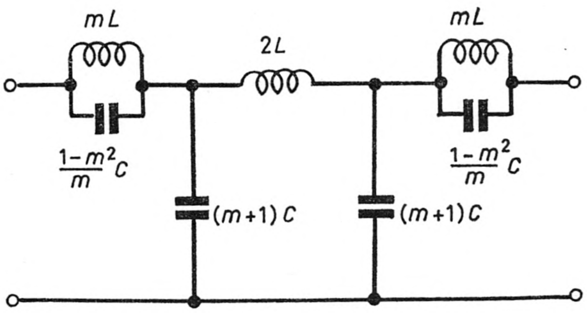


Fig. 3. Suitable low-pass filter for channels ($m = 0{\cdot}6$, L, C tune to 3·6 kHz).

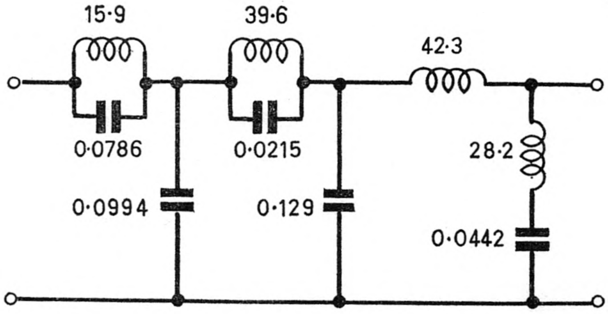


Fig. 4. Low-pass channel filter made for experimental system. Inductance values in mH, capacitance values in μF, design resistance 600 Ω.

metric weighting may be allowed; thus a reasonable filter design should give perhaps 20 dB attenuation at 4 kHz, rising to 40 dB at 4·4 kHz. The three-inductor filter shown in Fig. 3 is probably adequate. The four-inductor filter which was made for the experimental equipment (discussed later) was as shown in Fig. 4, but its measured attenuation/frequency response shown in Fig. 5 is rather better than is really needed.

At the sending end there will probably need to be an attenuator preceding the modulator in order to reduce the signal level to the 10 to 20 mV in 18 kΩ which is the maximum permissible for the constant-resistance modulator. If this is the case, the low-pass filter can probably precede the attenuator and be adequately decoupled from the modulator. If no attenuator is needed, or if its attenuation does not provide adequate decoupling, then a constant-resistance low-pass network should be used as shown in Fig. 2, with its constant-resistance port facing the modulator.

As indicated earlier, only one transformer is needed at each end. It should match the impedance of the line amplifier to that of six modulators in parallel; in the specific modulator design mentioned above this would be 3 kΩ. The loss per channel due to this parallelling of the modulators is $10 \log_{10} 6$ or approximately 7·8 dB. It is necessary that the tapped winding be wound as two twinned windings (as is usual in modulator transformers) to ensure a very low leakage inductance in the carrier path. The conversion loss of each modulator is about 5 dB.

In passing, it should be noted that each carrier supply must be quite pure, other tones being about 60 dB down if interchannel crosstalk from this cause is to be avoided.

3.2. *Other Causes of Interchannel Crosstalk*

There are several possible mechanisms for interchannel crosstalk in this system in addition to the second-order modulation of the input impedance of the modulators. Three mechanisms of which the author is aware are as follows:

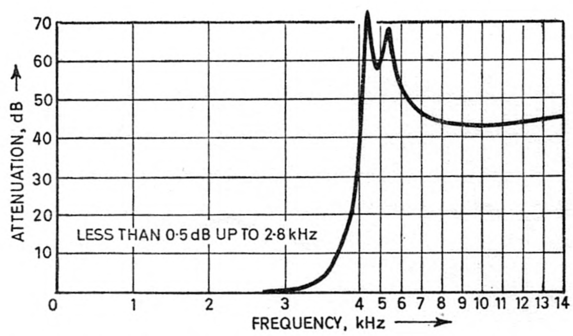


Fig. 5. Measured response of low-pass filter shown in Fig. 4.

(i) Harmonic sidebands may be produced in the sending-end modulators, for example if ω_p is the carrier frequency and ω_a an audio frequency, then $\omega_p \pm n\omega_a$ may be produced; if $n\omega_a$ lies outside the frequency band allocated to the channel, this product will fall within the band of another channel and will be demodulated with it, thus causing crosstalk.

(ii) Carrier leak at frequency $2\omega_p$ will be produced at the multiplexing bus-bars (or 'multiples') due to unbalance in the modulators. If this leak can be modulated by signals on the multiples, then products of the type $2\omega_p - \omega_q$ can be produced and cause crosstalk.

(iii) Carrier leak at frequency ω_p will also be produced at the multiples, and thus in the transmitted signal from each channel there will be a component corresponding to an envelope-modulated carrier, i.e. carrier plus two-side-bands. If any receiving modulator is suitably unbalanced, it can produce some rectification of this component, leading to its direct demodulation into a channel for which it is not intended.

Examining each of these causes in turn, we find that (i) is hard to calculate for the type of modulator being used here. For a ring modulator using ideal rectifiers, however, the calculation is straightforward,[6] and the voltage ratio of a third harmonic sideband ($\omega_p \pm 3\omega_a$) to the first-order sideband can be shown to be $x^2/24$, where x is the ratio of signal voltage to carrier voltage on the modulator elements. In our case x would be of the order of 0·1, rising to perhaps 0·2 on a very strong speech signal. This would give a harmonic-sideband/first-order sideband ratio of about 4×10^{-4} rising to perhaps 16×10^{-4}, or -68 dB rising to -56 dB. This does therefore appear to be a potentially serious source of crosstalk, but in the experimental system no crosstalk due to this cause could be detected at levels (relative to the wanted signal) down to -70 dB.

Mechanism (ii) is much more difficult to deal with. Experimentally it cannot be separated from the modulation of signal by the input resistance when it departs from the constant-resistance condition. Furthermore, all carrier leak and unbalance effects are intractable in theoretical calculations. A method of determining the distribution of the products of unbalance at both input and output ports of a ring modulator is given in Appendix 1, and this is useful in showing what products *can* occur at each port, and how they may be controlled by adjustment of potentiometers in the circuit; but it does not determine the relative amplitudes.

As far as can be determined from this approach, a voltage of $2\omega_p \pm \omega_q$ is to be expected at the input port of the receiving modulators (and at the output port of the sending modulators if ω_q is still taken as the single-sideband frequency at the multiplexing bus-bars) due to the unbalance in the modulator, and it will not respond to adjustment of the potentiometers. We know, of course, that this frequency is produced by departure from the constant-resistance condition, and it is possible that the two effects are the same when the latter is due to differences between one rectifier and another. Certainly, in all the experimental work reported on the constant-resistance modulator,[3,4,7] no dependence on balancing conditions was found for the $2\omega_p \pm \omega_q$ product.

From Appendix 1 it can be seen that unbalance can lead to the products $2\omega_p \pm 2r\omega_q$, where r is an integer, but by the same arguments as used for mechanism (i), with the need for unbalance in addition, it can be seen that the effect may be expected to be negligible in practice. No crosstalk due to this effect has been detected in the experimental system.

This leaves mechanism (iii), and unfortunately this one is not negligible. In the type of modulator being considered here, leak of the carrier frequency is likely to be as large as 3 mV if specially-matched rectifiers are not used nor special balancing carried out with potentiometers. This is only a few decibels below the sideband level. As far as rectification of the envelope-modulated component is concerned, none can take place in a perfectly-balanced modulator; but, with unbalance present, there is some rectification and the outputs of the receiving modulators will all contain some of the audio signal from other channels.

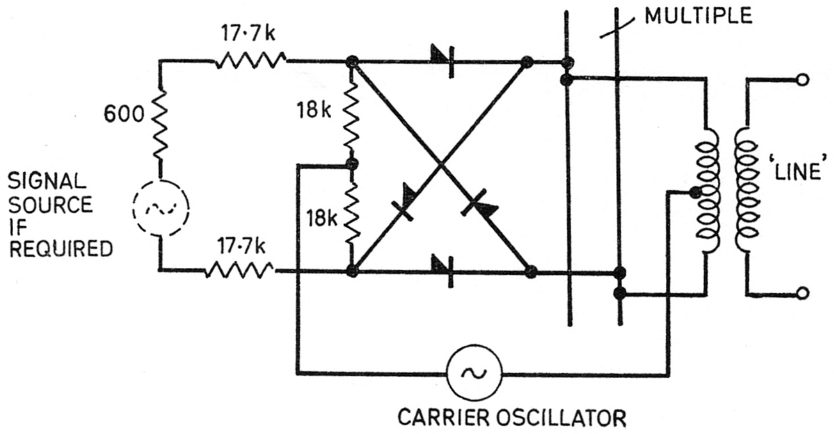
Once again, the calculation of the rectification is difficult for the actual type of modulator used, but if we consider instead the effect in a modulator using a single ideal or perfect-switch rectifier, we find it is essentially the same as the 'detector discrimination' effect in radio detectors.[8,9] Here the well-known result is that if the weaker carrier has an amplitude ratio of x relative to the stronger carrier, then the output voltage of the demodulated signal from the weaker carrier is $mx^2/4$ times the voltage of the stronger carrier, where m is the depth of modulation in the weaker signal. If we apply this to one of the rectifiers in our modulator, we can take x as 3 mV/100 mV, i.e. 0·03, and m as unity. This gives a demodulated output of about 0·2 mV. If we then assume the balance of the modulator is such that the opposing voltages on the two sides cancel to within 3%, the resultant demodulated output is about 0·006 mV. This is the crosstalk level at a point where the signal level is of the order of 6 to 10 mV. The crosstalk ratio is therefore about -60 dB. This is the order of crosstalk due to this cause actually measured in the experimental equipment.

3.3. *Practical Experience using the Constant-resistance Modulator*

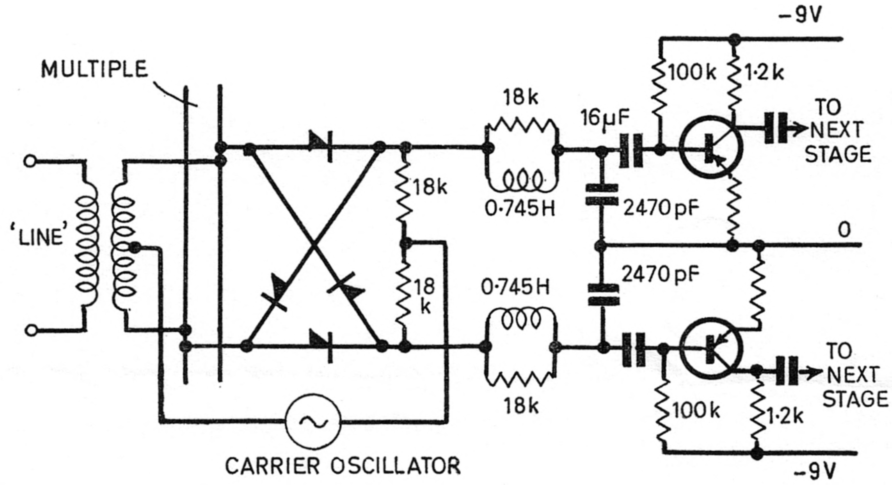
A six-channel experimental equipment was made according to the scheme of Fig. 2, with more detailed circuit information shown in Fig. 6. For tone tests, it was not necessary to use low-pass filters, and only one channel needed audio equipment, all others being terminated by a resistance at the audio side of the modulators. By interchanging carriers, all crosstalk paths could be tested. No line was used except that an attenuator was connected between transmit and receive multiples, and it was checked that no adverse effect occurred when all the attenuation was removed, leaving the multiples directly connected.

The carrier supplies were taken from six independent oscillators, each one supplying the appropriate modulators at sending and receiving ends. Since there was negligible phase-shift in the system between the modulators at the two ends, correct phasing was thus assured.

If the carriers had been harmonically related (e.g. by being harmonics of a common basic frequency), then crosstalk occurring by the second-order modulation of the modulator impedance and that due to mechanism (iii) mentioned in Section 3.2 would be



(a) Transmit end.



(b) Receive end.

Fig. 6. Details of experimental system.
(*Note.*—Each rectifier shown is two OA85's in parallel.)

indistinguishable, a tone of frequency f being received by both mechanisms in the channel suffering crosstalk when f was being transmitted on the sending channel; the only difference would be that mechanism (iii) would cause crosstalk into all channels and the second-order modulation would cause it only in one channel. But as only one channel was equipped for full reception, this slight distinction could not be used. Consequently, use was made of the independence of the carrier oscillators to cause the crosstalk outputs to be separated in frequency.

Consider a particular crosstalk test. A tone of 2·2 kHz was fed into channel 1 (64 kHz carrier). Second-order crosstalk was produced by the channel 2 modulators, the carrier frequency of which was $(72+\delta_1)$ kHz. Crosstalk was measured in channel 3 with a carrier frequency of $(80+\delta_2)$ kHz. Here δ_1 and δ_2 are arbitrary small frequency errors. The signal sidebands at the multiplexing bus-bars were 61·8 and 66·2 kHz. The former produced a crosstalk into channel 3 at $2{\cdot}2-(2\delta_1-\delta_2)$ kHz; the latter produced crosstalk at $2{\cdot}2+(2\delta_1-\delta_2)$ kHz. Crosstalk due to the rectification effect of mechanism (iii) was at 2·2 kHz since no frequency-translation was involved. Thus the separate causes could easily be distinguished and separately measured. Other tests were also made to check the mechanisms of crosstalk, e.g. by showing that the crosstalk at 2·2 kHz was independent of the 72 and 80 kHz oscillators, while the other crosstalk outputs were dependent on them.

It was found that there was comparatively little variation in crosstalk ratios from channel to channel, although the modulators had been made up under production conditions without special matching of the rectifiers. The crosstalk at 2·2 kHz, due to carrier leak and rectification, was consistently of the order of -65 dB relative to the wanted signal level, with carrier leak at the multiplexing bus-bars of the order of 10 dB below the sideband levels. It could be made worse by adding resistors in the modulators to deliberately unbalance them by large amounts. It was not very sensitive to signal level at the sending modulators, presumably because the component of envelope modulation was not affected much so long as there was a surplus of signal-sideband over carrier leak. An addition of 10 dB in the line loss reduced the crosstalk output by only 5 dB, and this seems more surprising.

The crosstalk ratio at $2{\cdot}2 \pm (2\delta_1 - \delta_2)$ kHz, due to second-order modulation in the modulator impedances, was consistently worse than that at 2·2 kHz, being typically -55 to -60 dB for each tone. Variations occurred over periods of time, presumably due to temperature variations (which were very considerable during the period of test, but were not recorded). Crosstalk ratios as bad as -45 dB were obtained. Such ratios were not at all sensitive to the level of the signal applied to the channels. If better crosstalk ratios are essential in a particular design, it would be wise to add some swamping (or decoupling) resistance at the modulator ports on the multiplexing side. For example, 6 kΩ resistors connected across each modulator port, or 1 kΩ across the multiple, with consequent rematching of the line transformer ratio, would improve crosstalk by 12 dB at the expense of 6 dB in the channel attenuation at each end.

When the carrier oscillators were set up carefully to be harmonically related, so that the various crosstalk outputs in each channel coincided in frequency, it was found possible to cause the two mechanisms of crosstalk to cancel out very substantially. The second-order crosstalk could not readily be controlled, but the addition of a potentiometer in the connection of the carriers to the centre of the transformer winding enabled carrier leak to be adjusted in magnitude and polarity, and resultant crosstalk ratios in the range -60 to -70 dB could easily be obtained. Such results were not, of course, very stable over a period of time and would hardly form a satisfactory basis for an operational system.

4. The Use of Ordinary Ring Modulators

Since one has to go to some trouble to obtain the constant-resistance condition, it is worth enquiring as to whether a modulator used at a more normal impedance level (say 600 Ω) and with more tolerance of higher signal levels could give an adequate performance.

Experimental measurements of the input modulation were made[7] on a ring modulator using the same OA85 rectifiers as the constant-resistance modulator, but with higher carrier voltages and terminating resistance of 600 Ω on the audio side. The relative level of the $(2\omega_p - \omega_a)$ product occurring at the input terminals when the signal at frequency $(\omega_p - \omega_a)$ was fed from a constant-current source is shown in the table below. Here $\hat{V}_c$ is the peak carrier voltage across the rectifiers, the carrier being fed from a high-resistance source so that the waveform of V_c was nearer to a square shape than a sinusoid.

$\hat{V}_c$ (volts)	0·5	1·0	1·5	2·0
Level of $(2\omega_p - \omega_a)$ relative to signal voltage (dB)	-19	-27	-29	-31

These results were found to be insensitive to signal voltage level up to at least 100 mV.

Use of a low-impedance carrier supply (which would be more economical of carrier power) would worsen these figures considerably, and although no systematic investigation was made using a low-impedance carrier, some rough tests suggested that the relative levels of $(2\omega_p - \omega_a)$ would not be better than -20 dB at any carrier voltage.

These figures lead to interchannel crosstalk of the order of -40 dB, which is of course unacceptable. It could be improved by inserting attenuators between the modulator and the multiplexing terminals, and this would be reasonably convenient in view of the higher signal levels that are usable with these modulators of low-impedance and high carrier voltage. Attention then needs to be paid to the carrier feed arrangements, since the use of a common return to the centre-point of the line transformer is no longer reasonable. A separate transformer for each modulator is really required, but this unfortunately adds to the cost.

The low-impedance arrangement discussed above is still, of course, an approximation to a constant-resistance modulator. The increased carrier voltage ensures that over a large part of the carrier cycle the rectifiers are either very low or very high resistances, so that if the load is a constant pure resistance at all frequencies, the input resistance is not grossly different from it. But if the modulator is used with no attempt at all to approximate to the constant-resistance condition, very much worse results are obtained. For example, it is shown in Appendix 2 that if a modulator is terminated at the audio end by a Zobel-type *filter* instead of the constant-resistance selective network shown in Fig. 1, the relative level of $(2\omega_p - \omega_a)$ is only about -12 dB.

The crosstalk due to carrier leak and rectification in these modulators tends to be worse than in the constant-resistance modulators because the much larger carrier voltages cause a disproportionately very much larger carrier leak. The insertion of attenuators between the multiplexing bus-bars and the modulators at the receiving end, however, has a beneficial compensating effect because it reduces the level of the signal relative to the carrier voltage of the receiving modulator. Experiments made using modulators as above suggest that crosstalk ratios of the order of -55 to -65 dB may be expected from this cause.

The carrier power required to drive the low-impedance modulators is, of course, vastly greater than that used in the constant-resistance system, and might be a source of considerable extra cost.

All in all, it seems that the constant-resistance modulator forms a much better basis for system design than the use of ordinary modulators.

5. The Use of Shunt Modulators

If constant-resistance modulators (preferably of the ring type) are used as recommended in the simple system of Fig. 2, then not only is the very undesirable second-order modulation avoided, but there is, in principle, no modulation at the multiplexing terminals of any kind whatever. If the system bandwidth is restricted to less than an octave, this last fact is of only secondary value, although it would become vital if larger bandwidths were required. Therefore, in the system of less than an octave, it is not strictly necessary to specify a constant-resistance modulator, but only a modulator which does not give second-order modulation at the multiplexing terminals. The shunt modulator (or for that matter, the series modulator) can be made[10] to produce no even-order modulation if

(i) the resistance/voltage law of the rectifiers is the same as is required for the constant-resistance ring modulator, namely

$$r(V_c) = a\exp(-bV_c)+r_0 \qquad \ldots\ldots(1)$$

over the relevant range of the carrier voltage V_c; here a, b and r_0 are constants;

(ii) the resultant circuit impedance (i.e. the signal source impedance and the load impedance in parallel for the shunt modulator or in series for the series modulator) is a constant resistance over time and at all relevant frequencies, such that its value R conforms to the equation

$$[r(+V_c)-r_0][r(-V_c)-r_0] = (R+r_0)^2 \qquad \ldots\ldots(2)$$

and

(iii) the carrier waveform must be the same on the backward half-cycle as on the forward; this can be achieved by connecting the corresponding modulators of another system to the carrier supply in opposite polarity.

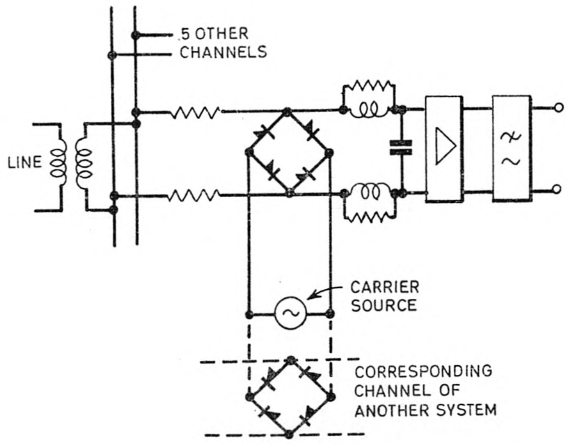


Fig. 7. Receiving-end circuit using shunt modulator.

It thus seems that the shunt (or series) modulator can be used in this application. It appears at first sight that condition (ii) may be spoilt by the fact that the other modulators, parallelled across the input or output of a particular channel modulator, provide a contribution to R which is varying with time. But the time variations of the other modulators contain no even-order harmonics of the carrier frequency, and are therefore not such as to spoil the operation of the one we are considering. There is, however, also an additional factor: it is not possible for the other modulator impedances connected in parallel with one another (in parallel with the line impedance too) to give the required resultant impedance at the level of $2R$, which when taken in parallel with an audio impedance of $2R$ would provide the resistance R required by eqn. (2). Thus a series resistance is needed in the connection between the multiplexing terminals and the modulator, giving the overall circuit schematic shown in Fig. 7. The series resistance produces about 10 dB loss in the channel, but of course gives 20 dB reduction in any second-order modulation which is produced due to errors in the rectifier law of eqn. (1). Since the shunt (or series) modulator has a conversion loss which is 6 dB greater than that of the corresponding ring modulator, it is evident that counting the series resistance the channel loss is very much greater than with the constant-resistance modulator in the scheme of Fig. 2. Clearly the system shown in Fig. 2 is preferable to that of Fig. 7.

6. Conclusions

Although it is apparent that there are several possibilities in the design of a f.d.m. system without band-pass filters, it is reasonable to conclude that the use of a properly-designed constant-resistance modulator is the best basis for a design of maximum economy.

7. Acknowledgments

The author wishes to record his thanks to Messrs. G. Terreault and J. R. Robertson, whose experimental work on the constant-resistance modulator formed the foundation for this study, and to Messrs. B. L. J. Kulesza and C. S. Nokes, who built and carried out the tests on the experimental six-channel system.

8. References

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9. Appendix 1:
Unbalance Effects at the Input and Output Terminals of Ring Modulators

Unbalance effects in modulators are difficult to calculate in terms of magnitudes, but a previous paper[11] has shown a basis for the deduction of the nature of the effects, and, in particular, for the use of potentiometers to reduce or eliminate leakage of particular products. The previous work was directed to the normal kind of modulators, and in considering the ring modulator was concerned only with leakage of unwanted frequencies into the output of the lattice. In the present paper we have seen that unbalance effects are important at the input as well as the output port. There are other applications too where input-port leak is important, e.g. the second-order ring modulator[12,13] where the normal output port of the lattice is terminated in an 'idler' load, and the required modulation-product output (at $2f_c \pm f_s$, where f_c is the carrier frequency and f_s is the signal frequency) is taken from the *input* port of the lattice.

We consider, for purposes of analysis, the ring modulator shown in Fig. 8. P1, P2, P3 and P4 are balancing potentiometers; R_1 and R_2 are provided to compensate for the mean resistance of P2 and P4. Since we are attempting to analyse only the nature and not the magnitude of the effects, several simplifications can be made. These are

(i) the circuit is wholly non-reactive and the transformers are ideal,

(ii) the signal and carrier sources have zero impedance (i.e. Z_s and $R_c = 0$),

(iii) the wanted products are taken as the current through a zero load impedance (i.e. $Z_L = 0$),

(iv) one-half of the signal voltage $V_s \cos 2\pi f_s t = 2b$ appears across each rectifier in spite of unbalances.

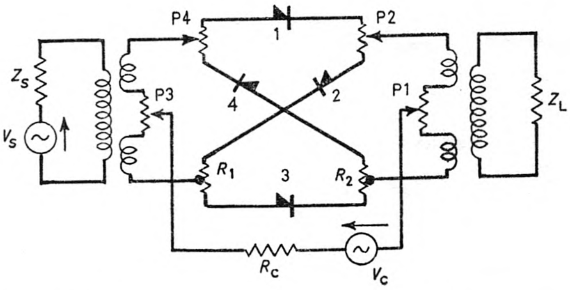

Fig. 8. Ring modulator with balancing potentiometers.

The carrier voltage $V_c \cos 2\pi f_c t$ is called a, and taking the polarity of currents and voltages in each rectifier relative to the direction of best conduction, the voltages across the four rectifiers 1, 2, 3 and 4 are respectively $(a+b)$, $-(a-b)$, $(a-b)$ and $-(a+b)$.

Let the current through a rectifier be related to the voltage across it by the power series

$$i = \sum_{n=0}^{\infty} d_n v^n \qquad \ldots\ldots(3)$$

In general the four rectifiers have different sets of coefficients d_{n1}, d_{n2}, d_{n3} and d_{n4}.

The current through the input terminals of the lattice is

$$
\begin{aligned}
i_1+i_2-i_3-i_4 &\\
&= \sum d_{n1}(a+b)^n+\sum d_{n2}(-1)^n(a-b)^n- \\
&\quad -\sum d_{n3}(a-b)^n-\sum d_{n4}(-1)^n(a+b)^n \quad \ldots\ldots(4) \\
&= \tfrac{1}{2}\sum[d_{n1}-(-1)^n d_{n2}+d_{n3}-(-1)^n d_{n4}]\times \\
&\quad \times[(a+b)^n-(a-b)^n]+ \\
&\quad +\tfrac{1}{2}\sum[d_{n1}+(-1)^n d_{n2}-d_{n3}-(-1)^n d_{n4}]\times \\
&\quad \times[(a+b)^n+(a-b)^n] \quad \ldots\ldots(5)
\end{aligned}
$$

Now a few trial expansions of $[(a+b)^n \pm (a-b)^n]$ for different values of n will quickly show that there are four groups of frequencies (where m and n are integers or zero):

(A) $2mf_c \pm (2r+1)f_s$ arising from $(a+b)^n-(a-b)^n$, n odd.

(B) $2mf_c \pm 2rf_s$ arising from $(a+b)^n+(a-b)^n$, n even.

(C) $(2m+1)f_c \pm 2rf_s$ arising from $(a+b)^n+(a-b)^n$, n odd.

(D) $(2m+1)f_c \pm (2r+1)f_s$ arising from $(a+b)^n-(a-b)^n$, n even.

Group (A) clearly includes the unwanted products $2f_c \pm f_s$ and has amplitude

$$d_{n1}+d_{n2}+d_{n3}+d_{n4} \quad \ldots\ldots(6)$$

which of course cannot be brought to zero by adjustment of any of the potentiometers, although it does reduce to zero in the constant-resistance condition.

Group (B) can be balanced out if we can obtain

$$d_{n1}+d_{n2}-d_{n3}-d_{n4}=0 \quad \ldots\ldots(7)$$

and this can be achieved (for a particular value of n) by adjustment of either P1 or P4.

Group (C) can be balanced out if we can obtain

$$d_{n1}-d_{n2}-d_{n3}+d_{n4}=0 \quad \ldots\ldots(8)$$

and this can be achieved by adjustment of P2 or P3.

Group (D) can be balanced out if we can obtain

$$d_{n1}-d_{n2}+d_{n3}-d_{n4}=0 \quad \ldots\ldots(9)$$

and this can be achieved by adjustment of P2 or P4.

To a first approximation only three of the four potentiometers are needed to give a zero current at the greatest possible number of particular frequencies from Groups B, C and D since there are only three independent eqns. (7, 8 and 9) to be satisfied. Only if the appropriate condition (7), (8) or (9) can be met simultaneously for all values of n can all frequencies in a group be balanced out, and this is clearly unlikely with practical rectifiers; but as the differences between rectifiers are likely to be greater in the scale of the current/voltage characteristic than in its shape, one would expect some tendency for the unbalance currents at all frequencies in a group to be small as one of them is brought to zero by a potentiometer adjustment. This expectation is supported by the experimental observations reported for the currents at the output of the lattice in reference 11.

Experience (as e.g. in Fig. 4 of Reference 11) suggests that those leakage or unbalance currents of frequencies which involve f_s (i.e. all product frequencies other than harmonics of the carrier) have usually-negligible magnitudes, very much smaller than the carrier leak multiplied by V_s/V_c; and this is largely due to the fact that in practical modulators (as compared with the simplified model used above) the signal voltage appearing across the rectifiers is only a fraction of the signal voltage applied to the modulator owing to the effect of a finite load impedance. Thus in the second-order ring modulator the unwanted current at frequency $f_c \pm f_s$ (for example) at the input of the lattice is very small compared with the wanted current at $2f_c \pm f_s$. In the constant-resistance modulator, where ideally no modulation product frequencies occur at the input, it is normally departures from the constant-resistance condition which cause $2f_c \pm f_s$ to appear there and this has been found in experiments [3,7] to be the dominant effect.

Table 1

Summary of effects of potentiometers (m and r are integers or may be zero)

Potentio-meter	Frequencies controlled at input of lattice	Frequencies controlled at output of lattice
P1	$2mf_c \pm 2rf_s$	$(2m+1)f_c \pm 2rf_s$
P2	$(2m+1)f_c \pm 2rf_s$ $(2m+1)f_c \pm (2r+1)f_s$	$2mf_c \pm 2rf_s$ $2mf_c \pm (2r+1)f_s$
P3	$(2m+1)f_c \pm 2rf_s$	$2mf_c \pm 2rf_s$
P4	$2mf_c \pm 2rf_s$ $(2m+1)f_c \pm (2r+1)f_s$	$(2m+1)f_c \pm 2rf_s$ $2mf_c \pm (2r+1)f_s$

Finally the results of both this and the previous examination[11] of the effects of the potentiometers are summarized in Table 1. This is a useful comparison of the effects at the input and output of the lattice, for it shows, among other things, that carrier leak (i.e. f_c and harmonics of f_c) can be controlled by P1 to give a good balancing-out of f_c at the output of the lattice or of $2f_c$ at the input; and often one balancing adjustment will give low leaks of both simultaneously. Similarly P3 will deal with d.c. and $2f_c$ at the output and f_c at the input. P1 and P3 are the balancing potentiometers normally used in modulators, since carrier leak is usually the most serious unbalance product. The use of P2 and P4 is unusual, but one or other is essential if leakage of the input signal into the output of the lattice is to be eliminated, or leakage of $f_c \pm f_s$ into the input.

10. Appendix 2:

Second-order Modulation in Ring Modulators Terminated by a Filter

The problem considered here is the calculation of the second-order modulation at the input terminals of the modulator when its output is connected directly to a low-pass filter of normal type (e.g. Zobel image-type filter). Such a filter has an image impedance which is a frequency-dependent resistance in the pass-band and a pure reactance in the stop-band. The actual input impedance of the filter is only the same as the image impedance when the filter is image-matched at its output, but clearly the input impedance departs very greatly from the constant resistance needed to terminate the modulator used in the constant-resistance mode of Fig. 2.

If we take as an example the extreme case when the input impedance is the image impedance, and then take this to be a pure resistance at the wanted audio frequency $\omega_p - \omega_q$ (where ω_p = carrier and ω_q = frequency at the multiplexing bus-bars) and a zero† reactance (i.e. short-circuit) at all other output sideband frequencies, we find that, instead of being a constant-resistance modulator with negligible even-order modulation at the input terminals, the modulator is in fact a highly-efficient even-order modulator giving even-order products at the input terminals only a few decibels below the applied signal. Indeed, a very similar arrangement has recently been put forward and investigated as an advantageous modulator for second-order modulation.[12,13] Adapting the analysis of Reference 13, the actual value of the ratio of the currents at ω_q and $2\omega_p - \omega_q$ at the input is calculated below with some specified assumptions.

We make the following assumptions:

(*a*) the rectifiers are ideal switches without resistance (i.e. forward resistance = 0, backward resistance = ∞);

(*b*) the output of the modulator is terminated with a filter which effectively gives a termination of R_1 at the audio frequency $\omega_p - \omega_q$ but zero at all other relevant frequencies;

(*c*) the signal source has a resistance R at all frequencies.

† If the filter has the other type of image impedance, so that 'zero' should here be replaced by 'infinite', then the whole of the results apply just the same provided the ratio R_1/R is inverted to R/R_1.

These differ from the conditions considered in Section 4 of Reference 13 only in respect of the substitution of the resistance R_1 for a finite reactance. The analysis of Reference 13 can thus be easily adapted, and with $V \cos \omega_q t$ representing the applied signal, eqn. (48) of Reference 13 becomes for the present case:

$$V - [R + R_1(4/\pi^2)]i_0 = R_1(4/\pi^2)\left(\frac{\pi^2}{4} - 1\right) i_2 \qquad \text{......(10)}$$

where i_0 is the input current at frequency ω_q and i_2 is the current at the input terminals at frequency $2\omega_p - \omega_q$. Also eqn. (39) of Reference 13 gives us for the present case

$$\frac{i_0}{i_2} = -\frac{R + R_1(4/\pi^2)\left(\frac{\pi^2}{4} - 1\right)}{R_1(4/\pi^2)} \qquad \text{......(11)}$$

The ratio of the currents, i_0/i_2, is probably of main interest here, but the use of eqn. (11) applied to eqn. (10) will also enable us to obtain the ratio of the voltages at ω_q and $2\omega_p - \omega_q$, i.e. $(V - Ri_0)/Ri_2$.

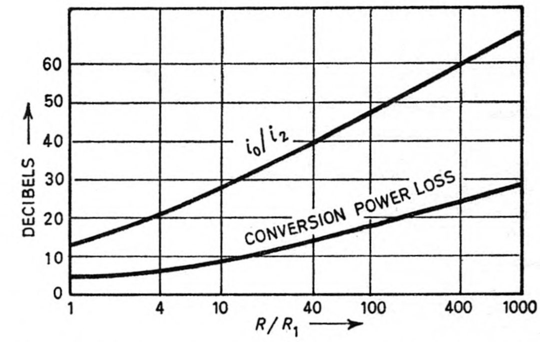

Fig. 9. Graph showing second-order modulation in a ring modulator terminated in an image-type low-pass filter, together with conversion power loss, plotted as functions of R/R_1, where R is the nominal circuit impedance before the modulator and R_1 is the pass-band image impedance of the filter.

The ratio i_0/i_2 expressed in decibels is plotted in Fig. 9 as a function of R/R_1. One would suppose a normal choice of R_1 would be to make it equal to R. The second-order interference current i_2 is then only 12 dB below the input current, and this is the figure which has already been quoted in the paper.

As R_1 is increased, this ratio worsens to a limit of 1·67 dB when R_1 becomes infinite. As R_1 is decreased, however, the ratio improves, and the desired ratio, which we may take to be 60 dB, is attained when $R_1 = R/400$. It is thus important to know what happens to the conversion loss of the modulator when

R_1 is made very small, as this might be an alternative to the use of decoupling circuits between the multiplexing bus-bars and the modulator.

The relevant measure of the conversion loss is the conversion-power loss (c.p.l.) ratio defined as:

$$\text{c.p.l. ratio} = \frac{\text{power available from signal source}}{\text{power in load at } \omega_p - \omega_q} \qquad \ldots\ldots(12)$$

$$= V^2/4RR_1 i_1^2 \qquad \ldots\ldots(13)$$

where i_1 is current at frequency $\omega_p - \omega_q$. From eqn. (13), therefore,

$$\text{c.p.l. ratio} = \frac{\pi^2}{4} \cdot \frac{(R+R_1)^2}{4RR_1} \qquad \ldots\ldots(14)$$

This ratio, expressed in decibels, is also plotted in Fig. 9, and will be seen to increase as R_1 decreases. At the value $R_1 = R/400$ which was required to reduce the interference to 60 dB below the signal, the conversion power loss is 24 dB. The difference in dB between the interference ratio and the c.p.l. ratio is seen from eqns. (11) and (14) to approach the value

$$20 \log_{10} \pi\sqrt{R/R_1} \quad \text{dB} \qquad \ldots\ldots(15)$$

as R_1 becomes very small compared with R.

It should be remembered that the above calculations have assumed that the rectifiers operate as perfect switches. When used in the exponential region (as in the constant-resistance design considered in this paper) with a reasonable carrier voltage, so that a high ratio of maximum to minimum resistance is obtained, and with normal terminating impedances, the conversion loss is increased by between 1 and 2 dB, and the interference current is reduced by perhaps 2 to 3 dB. But with the extremely low values of R_1 discussed above, this difference is probably greatly increased.

It can be seen that this method of reducing the second-order modulation at the multiplexing bus-bars has exactly the same effect on the overall attenuation in the channel as the decoupling method discussed in the paper.

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