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Arrays with Constant Beam-Width over a Wide Frequency-Range

In many sound-ranging and listening systems (principally used under water) it is necessary to receive signals over a wide range of frequency, the ratio of upper to lower frequency being of the order of ten or more. Transducers (or hydrophones) with a sufficiently wide frequency response are available, but usually have to be used as omni-directional receivers because otherwise their beam-width would be inversely proportional to frequency. Clearly, the use of directional receivers would be very advantageous in many applications, but an essential condition is that the beam-width must be constant over the appropriate range of frequencies. An approach to the solution of this problem is to connect the outermost units of the transducer to the receiving amplifiers through low-pass filters, so that the transducer is virtually made smaller at the higher frequencies. There are, however, many difficulties and complications in this process, which is, at best, rather crude.

A more elegant solution of the problem, and one in which the design problems are more straightforward, can be applied in the case of strip (or line) transducers, which are more suitably referred to as arrays of individual small transducers arranged in a

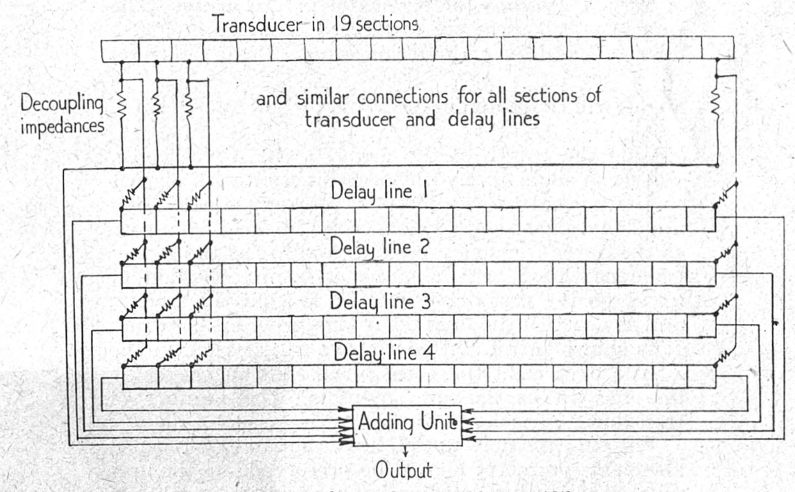

Fig. 1. General scheme of constant beam-width array

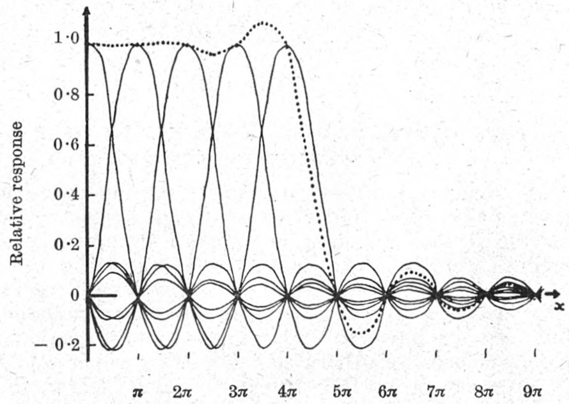

Fig. 2. Right-hand half of beam patterns at upper frequency limit for $r = 4$; $a_m = 1$ for all values of m. Component patterns, ———; resultant pattern, ... It is assumed that the number of sections in the array is large

straight line. Directionality is achieved in all planes containing the line of the array. The solution is based on the synthesis[1-3] of directional patterns from elementary $(\sin x)/x$ patterns using delay lines to produce these patterns at various angles of deflexion from the normal. The basic idea is that when the beam-width between half-power points is to be constant over a frequency range of ratio $2r + 1$, then $2r + 1$ patterns are superposed. One of these is the straight undeflected pattern of the array and the remainder is the set of patterns obtained from the $2r$ outputs of r delay lines. There must be a minimum of $2r + 1$ sections (or elements) in the array. The way in which these are connected is shown in Fig. 1 for $r = 4$ and for a 19-section array. Beam deflexion angles will be quoted in terms of x, where $x = \frac{l\pi}{\lambda} \sin \theta$, l being the length of the array, λ the wave-length, and θ the angle of any physical direction with respect to the normal to the line of the array. Each delay line has a phase-shift which varies linearly from zero at the lower frequency limit to a value at the higher frequency limit which causes a beam deflexion of π rad. on the first delay line, 2π rad. on the second, and so on until the final delay line, where it is $r\pi$ rad. It is shown in ref. 1 that to give a deflexion on the x-scale of π rad., the total phase-shift in the delay line has to be 2π rad., assuming the number of transducer sections in the array is large.

The patterns need not all have the same amplitude, although symmetry should be preserved. If the outputs of the mth delay line have a relative amplitude a_m, then it is clear that at the lowest frequency, where the phase-shifts are zero, the overall pattern is merely

$$\left[\sum_{m=-r}^{m=+r} a_m\right] \cdot \frac{\sin x}{x}$$

As the frequency increases, so the patterns derived from the delay lines are spread out from the axis of the beam, thus widening the beam on the x-scale at the same rate at which the x-scale itself increases relative to the physical angles (θ) due to the decreasing wave-length (λ). A constant beam-width thus tends to be maintained. (An increase of frequency beyond the design maximum will separate the patterns so that dips occur in between them.) If we take the minimum frequency to have unity angular velocity, so that the highest frequency has the angular velocity of $2r + 1$, then at any frequency the directional pattern is

$$D(x) = \sum_{m=-r}^{m=+r} a_m \cdot \frac{\sin [x - m(\omega - 1)\pi/2r]}{[x - m(\omega - 1)\pi/2r]}$$

If $a_m = 1$ for all values of m, then a very flat-topped beam is obtained at the upper frequency, as illustrated in Fig. 2; and an almost exactly constant beam-width (measured between the half-power points) and a fairly constant directivity index are maintained. To maintain a reasonably constant shape of beam—as well as a fairly constant width—over the frequency range, a_m should diminish as m increases; but if the shape is to be maintained outside the half-power beam-width, then additional delay lines will be needed to give additional component patterns of greater angular deflexion.

The method may well also have applications in radio aerials.

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