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The Presentation of Electrical Network Theory

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SUMMARY

A new introductory course in electrical network theory is outlined, with a discussion of its more critical features. The intention of the course is to ensure a better understanding of the subject by the students, rather than to alter the traditional coverage; and this is done by arousing the students' interest and anchoring their thoughts to some simple and practical conceptions which are used throughout.

1. INTRODUCTION

The usual courses in electrical network theory, whether in universities or technical colleges, appear, so far as they are known to the author, to be open to serious criticism on a number of grounds, chief among which are:

(a) Excessive abstraction in the early stages—e.g. the emphasis on the formal network simultaneous equations (mesh or nodal equations) and the insistence on the use of determinants.

(b) Excessive emphasis on circuits which obscure the simpler conceptions—e.g. the use of the circuit of Fig. 1(a) in the initial discussions on resonant circuits, when the circuit of Fig. 1(b) is much easier to deal with.

(c) The use of terms which are meaningless, and therefore confusing, to the student at the stage at which they are introduced—e.g. "decrement" and "time-constant" when only steady-state analysis is covered.

(d) The use of difficult parameters, such as "image attenuation", at too early a stage, and the making of simple parameters such as "insertion loss" difficult.

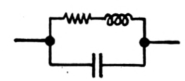

FIG. 1a

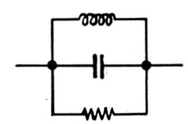

FIG. 1b

The results of such treatments appear to be that in lower technological courses, where the students tend to be examined on what they *know*, good marks are obtained; but in higher courses, where students are tested for their *understanding* and ability to extend their knowledge by their own thought, very poor results are obtained. In other words, such treatments are not difficult to learn, but do not very greatly assist the understanding of network theory and performance.

When the author first went to the University of Birmingham, the subject of networks was taught by a most competent network expert, who was also a good lecturer, and yet the subject was causing more failures than any other. The time was obviously ripe for experiment, and the author introduced a new introductory course of about 34 lectures which is now given to all Electrical Engineering students in their first 4 terms of the Honours course (which is of 9 terms in all); and the first 20 lectures (together with a further 8 special lectures) are given also to those Physics students who choose Electrical Engineering as a subsidiary subject. The course has been given four times so far, to about 160 students in all, and the results can be said to be very good. Not only are examination results very much better, but the students appear to proceed to the advanced courses in networks with more confidence.

The basic principle of the new course is to arouse the student's interest, and to anchor his thoughts to some secure and simple conceptions, by starting with the idea of "insertion loss" of a four-terminal (or two terminal pair) network, and by considering the most common practical problem—that of designing networks to operate in cascade in a long electrical system. For this latter purpose, the iterative network is an obvious choice, and leads to a very simple analysis of ˥, T, Π, bridged–T, and lattice networks, all containing only resistances in the first instance. The equivalence of T and Π networks is taken in one's stride and is used to reduce complicated networks to simple ones. In the analysis of lattice networks, the opportunity is taken to introduce mesh and nodal analysis and to develop the general network equations. The usual networks theorems are worked in also. The course then proceeds to a discussion of reactive circuits, resonance, Foster's Reactance Theorem, constant-resistance networks, loss and phase equalizers and some elementary ideas on synthesis. Only after all this is the "image" conception brought in.

This basic course is followed by a parallel presentation of filter design and synthesis, transformers, transmission lines, etc. on the one hand, and transient analysis and operational calculus on the other.

In the hope that it will be of interest and assistance to others, an outline of the basic course is given below. Comments and criticisms will be welcomed. Discussion has been restricted to those matters which are not part of the traditional courses or are not dealt with in elementary textbooks, with the result that the space devoted here to the various topics is no indication of the relative time devoted to them in teaching.

2. RESISTANCE NETWORKS (13 lectures)

2.1. DEFINITIONS

We start with a description of 4-terminal networks, and illustrate the way they are built up in cascade into a system, such as a tele-

phone transmission system. (Other numbers of terminals are encountered, but apart from 2-terminal networks, the 4-terminal network is by far the most common*). It is generally a requirement of such systems that the insertion loss of each network is to have prescribed values at given frequencies, and that the whole system is designed in such a way, known as the iterative method, that the insertion loss of each network is unaffected by the networks on each side of it—only in this way can the overall loss be readily calculated.

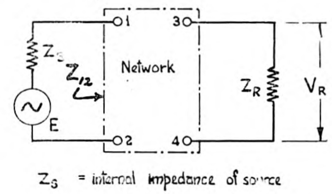

FIG. 2.

(i) *Definition.* INSERTION LOSS

Fig. 2 shows the circuit conditions.

$$\text{Insertion Loss Ratio, } A_L = \frac{V_R'}{V_R} \qquad \ldots\ldots\ldots\ldots (1)$$

where V_R is the output voltage when the network is inserted, and V_R' is the output voltage when the network is removed.

(It is as well to mention at this stage that, in general, Z_S, Z_R, and the network may have reactance, and that the insertion loss in $db = 20 \log_{10} \left| \frac{V_R'}{V_R} \right|$ and the insertion phase-shift $= \arg V_R' - \arg V_R$).

(ii) *Definition.* ITERATIVE OPERATION

This is when every network has an input impedance Z_{12} (when the network is terminated by Z_R on 3, 4†) equal to Z_R. In these circumstances, a particular network is always terminated by Z_R whether it is the last of the system or is followed by other networks.‡ It is also easily shown that in these circumstances the insertion loss is independent of Z_S, i.e. of the circuit preceding. Thus, since

$$Z_{12} = Z_R, \text{ then } V_R' = V_{12} \text{ and } A_L = \frac{V_{12}}{V_R} \qquad \ldots\ldots\ldots\ldots (2)$$

The ratio V_{12}/V_R is clearly a function only of the way the voltages and currents are divided *after* the terminals 1, 2.

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*The term "3-terminal network" is not used in this course, to avoid confusion.

†If the use of numerical subscripts is thought to be confusing in relation to their later usage for self- and mutual-impedances, etc., then the terminals can be labelled *a*, *b* and *c*, *d*, and letter subscripts used.

‡It is probably only confusing to introduce at this stage the idea of networks which are iterative with respect to the source impedance.

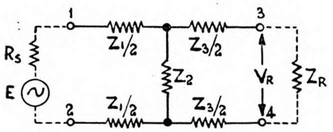


FIG. 3 a

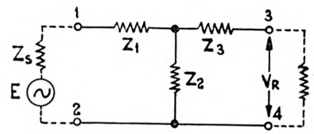


FIG. 3 b

So it is concluded that with iterative operation, the insertion loss of each network is unaffected by its whereabouts in the system. The overall insertion loss ratio of the whole system is the product of the insertion loss ratios of the individual networks.

(iii) At this first stage it is also desirable to discuss balanced and unbalanced networks, and to show that, in most practical situations, the two networks of Fig. 3, for example, are indistinguishable as far as transmission from 1, 2 to 3, 4 is concerned, the voltage V_R being the same in both circuits.

2.2. TWO-ELEMENT NETWORKS

We now confine ourselves to pure resistances until the basic network configurations have all been introduced. It is easily explained, now and later, that R may be replaced by Z in all equations, provided it is remembered that Z is complex. (A lot depends, of course, on the existing knowledge of the students. University students in their first term can*not* be assumed to know about impedances and the j operator).

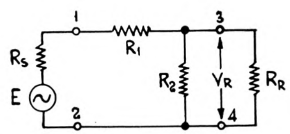


FIG. 4 a

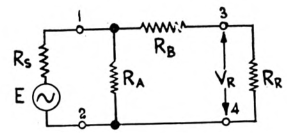


FIG. 4 b

(i) The simplest networks possible are now analysed—i.e. those of Fig. 4. By considering the network of Fig. 4(a)—which we call the ˥-network—as a potentiometer, it is easily shown that for this network :

$$A_L=\frac{V_R'}{V_R}=1+\frac{R_S\,R_R+R_1\,R_R+R_1\,R_2}{R_2\,(R_S+R_R)} \quad \ldots\ldots\ldots (3)$$

This form demonstrates that the value of A_L ranges from unity upwards. For a given A_L, there is an infinite number of pairs of values of R_1 and R_2.

(ii) Specifying now that the network is to be iterative (and that its "iterative resistance" is to be R_R) we obtain a second equation

$$R_{12}=R_1+\frac{R_2\,R_R}{R_2+R_R}=R_R$$

giving $R_1\,R_2+R_1\,R_R-R_R^2=0$ .... (4)

Solving (3) and (4) gives

$$A_L=1+\frac{R_R}{R_2}=\frac{R_R}{R_R-R_1} \quad \ldots\ldots\ldots (5)$$

which is, as we would expect, independent of R_S, and

$$R_1=R_R\cdot\frac{A_L-1}{A_L} \text{ and } R_2=\frac{R_R}{A_L-1}. \quad \ldots\ldots\ldots (6)$$

(iii) It is worth demonstrating that A_L is *not* independent of R_S when the network is not iterative. Students at Birmingham do a laboratory experiment on this.

(iv) The network of Fig. 4(b)—the Γ-network—can be similarly treated, and gives, for iterative operation :

$$R_A=R_R\cdot\frac{A_L}{A_L-1} \text{ and } R_B=R_R\,(A_L-1) \quad \ldots\ldots\ldots (7)$$

This is a suitable point at which to introduce the idea that some networks have the same equations as others provided that resistances in one group are replaced by conductances in the other.

Thus if $G=1/R$, eqn. (7) becomes

$$G_A=G_R\cdot\frac{A_L-1}{A_L} \text{ and } G_B=\frac{G_R}{A_L-1} \quad \ldots\ldots\ldots (8)$$

which is of exactly the same form as eqn. (6). The idea is not fully developed, however, until T– and ΠΠ–networks are reached.

(v) The equivalence of ˥–and Γ–networks for certain purposes is now discussed. As these networks have each only two variables, they can be equated in respect of only two independent characteristics. They can, for example, be made to be iterative with respect to the same R_R and then to have the same A_L. It is easily shown that they then have completely different values of R_{34}, i.e. the resistance looking back into the networks.

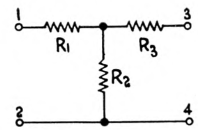


FIG. 5 a

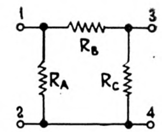


FIG 5 b

2.3. T– AND ΠΠ–NETWORKS

The notation is shown in Fig. 5.

(i) Simple algebra is used to derive the equations for the T–network : e.g.

$$A_L=\frac{R_S\,R_R+R_S\,R_2+R_S\,R_3+R_R\,R_1+R_R\,R_2+R_1\,R_2+R_2\,R_3+R_1\,R_3}{R_2\,(R_S+R_R)} \quad \ldots\ldots\ldots (9)$$

This is cumbersome, and little is to be gained by using the unsymmetrical network in the present context. It is best to take $R_1=R_3$ and work out the conditions for iterative operation. Since the network now has the same iterative resistance looking both ways through it, the term "characteristic" resistance (or impedance)" can be introduced here, if desired. Following the pattern of 2·2 (ii), we obtain for the symmetrical iterative network :

$$A_L = \frac{R_R + R_1}{R_R - R_1} \quad \ldots\ldots\ldots\ldots (10)$$

from which, of course, R_S has dropped out, and

$$R_1 = \frac{A_L - 1}{A_L + 1} \cdot R_R \text{ and } R_2 = \frac{2A_L}{(A_L - 1)\,(A_L + 1)} \cdot R_R \quad \ldots\ldots\ldots\ldots (11)$$

(ii) The Π–network is treated similarly, *but using conductances* throughout, and identical equations are obtained apart from the replacement of resistance by conductance ; e.g.

$$G_A = \frac{1}{R_A} = \frac{A_L - 1}{A_L + 1} \cdot G_R \text{ and } G_B = \frac{1}{R_B} = \frac{2A_L}{(A_L - 1)\,(A_L + 1)} \cdot G_R \quad \ldots\ldots (12)$$

for the iterative condition.

(iii) The idea of "dual" networks is now discussed in terms of the replacement of resistanc n series by conductance in shunt and vice versa, and resistance in shunt by conductance in series and vice versa. This is a very limited aspect of duality, however, and it is best not taken too far at this stage.

(iv) The equivalence of T–and Π–networks is now discussed, and the usual equations derived for conversion from one to the other, using open- and short-circuit resistances at input and output terminals as the basis of equation. This is standard text-book work and so is not dealt with further here, except to point out that it is a very worthwhile exercise for the students to prove that the networks are equivalent in every way they can think of, e.g. the resistance R_{13} (1, 2 and 3, 4 open-circuit) is the same for both.

(v) The reduction of complicated 4-terminal networks to simple T- or Π–form, using T to Π and Π to T conversions, is dealt with by means of numerous examples. The "Bridged–T" and "Twin–T" networks are also dealt with by this method, and a demonstration given of how much quicker and simpler it is in these cases than direct methods of analysis. It is made clear that not all networks can be reduced in this way.

(vi) Finally in this section, we make sure that the student is clear about insertion loss by considering the case of an unsymmetrical network and showing that the insertion loss is, even then, the same in both directions of transmission through the network, provided the terminating conditions are not changed.

2.4. NETWORK THEOREMS

It is convenient to discuss the usual network theorems at this stage. Rigorous proofs cannot be given on the basis of the work so far covered in the course—nor indeed would they be desirable at so early a stage. The student can be referred to a good text-book

for them if desired. But concise demonstrations of the reasonableness of the theorems are highly desirable, and are indicated below.*

(i) *Four-terminal network theorem.* "Any network of impedances, however complicated, having a pair of input and a pair of output terminals, can have its external performance in relation to these pairs of terminals fully specified by three external parameters". Actually, four-terminal networks fall into two classes :

(A) Those which can be fully specified in *all* external respects by three external parameters, and

(B) Those which can be specified by three external parameters only in relation to their use between two pairs of terminals. This class is properly called "two terminal pair" networks.

Into class A fall all networks which have not more than three elements (for less than 3 elements the external parameters are not independent), and all those which can be reduced by T to Π and Π to T conversions to simple T–or Π–networks.

Into class B fall all other four-terminal networks. It is only for this class that the truth of the theorem is not obvious. An example of a network of class B is shown in Fig. 6(a), where, in general, $R_2 \neq R_3$. For transmission between terminal pairs 1, 2 and 3, 4 this network is clearly exactly equivalent to that of Fig. 6(b), which is a 3-element network. Not all possible networks are so easily dealt with, however ; an unsymmetrical lattice is very resistant to such a kind of argument.

(ii) *Thevenin's theorem.* This is standard text-book work. The proof uses the four-terminal network theorem by substituting for

*If these theorems were postponed until after Section 2.5 (iv), it would then be possible to derive them from the determinantal solutions of the general network equations. The four-terminal network theorem, for example, follows from these in the form of the open-circuit impedance equations

$$Z_{11}i_1 + Z_{12}\,i_2 = e_1$$
$$Z_{21}i_1 + Z_{22}i_2 = e_2$$

and the short-circuit admittance equations

$$y_{11}e_1 + y_{12}e_2 = i_1$$
$$y_{21}e_1 + y_{22}e_2 = i_2$$

where it is shown that $Z_{12} = Z_{21}$ and $y_{12} = y_{21}$ (i.e. reciprocity theorem). But the author feels that the idea of transfer impedance and transfer admittance is far too abstract for the students at this stage, and should not be introduced until the more advanced course is reached.

Note also that since the short-circuit impedances are

$$\frac{1}{y_{11}} = \frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{22}}$$

and $$\frac{1}{y_{22}} = \frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{11}}$$

it is evident that the use of open- and short-circuit impedances as a basis of obtaining equivalent networks [Section 2.3 (iv) and 2.5 (v)(B)] is essentially the same as the use of open-circuit driving-point and transfer impedances. There is, however, this important difference ; in the short-circuit impedance ($^1/y_{11}$) the transfer impedance (Z_{12}) occurs only as Z_{12}^2. Therefore, reversal of polarity in a lattice network, which makes Z_{12} negative, is not implicitly taken account of in the open- and short-circuit impedances, causing the difficulty mentioned in Section 2.5 (v) (B),

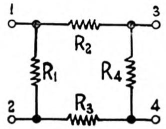


FIG. 6a

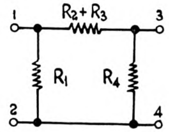


FIG. 6 b.

the unknown network an e.m.f. feeding the external load via a T–network.

(iii) *Norton's theorem.* Proved by the application of Thevenin's theorem.

(iv) *Superposition theorem.* No proof is offered. The application of Thevenin's and Norton's theorems when there are many generators in the unknown network is now discussed.

(v) *Reciprocity theorem.* Proved by use of the four-terminal network theorem. The e.m.f. and the current which are the subject of reciprocity are linked by a T–network which is considered to be equivalent to the real network whatever its configuration. The equation for E_1/I_2 is symmetrical and so proves the theorem.

(vi) *Compensation theorem.* Although this is given to the students, its value is not rated highly.

(vii) *Maximum Power-Transfer theorem.* Stated in the usual form, but proved only for pure resistances.

(viii) *Kirchhoff's Laws.* These are already known to the students, but are repeated here for completeness.

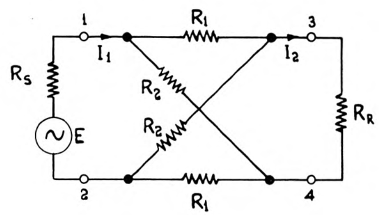


FIG. 7

2.5. LATTICE NETWORKS AND FORMAL NETWORK EQUATIONS

(Fig. 7 refers).

It is explained that only symmetrical lattice networks are generally encountered, and our treatment is confined to them. The fact that lattice networks are not amenable to direct solution by voltage-dividing techniques is made the excuse for discussing mesh and nodal analysis.

(i) Mesh analysis is used to solve the lattice network in the usual way, starting with Kirchhoff's mesh law. Determinantal solution of the three simultaneous equations is used first and its general importance stated ; but then direct elimination is shown to be

simpler for such a simple circuit as the lattice. The insertion loss ratio is shown to be

$$A_L=\frac{(R+R_1)\ (R+R_2)}{R\ (R_2-R_1)} \qquad \ldots\ldots\ldots\ (13)$$

when $R_S=R_R=R$. The important significance of the denominator term (R_2-R_1) in relation to polarity reversal is discussed. The lattice as a Wheatstone Bridge is conveniently discussed here.

(ii) The design of the lattice as an iterative network is now dealt with. The mesh equations are solved for the input current I_1 as well as for the output current I_2, giving

$$I_1=\frac{E\ (2R_R+R_1+R_2)}{(R_S+R_1)\ (2R_R+R_1+R_2)+(R_2-R_1)\ (R_R+R_1)} \qquad \ldots\ldots\ldots\ (14)$$

$$\text{and } I_2=\frac{E\ (R_2-R_1)}{(R_S+R_1)\ (2R_R+R_1+R_2)+(R_2-R_1)\ (R_R+R_1)} \qquad \ldots\ldots\ (15)$$

Now the input resistance is

$$R_{12}=\frac{E}{I_1}-R_S \qquad \ldots\ldots\ldots\ (16)$$

$$=\frac{R_1\ R_R+R_2\ R_R+2R_1\ R_2}{2R_R+R_1+R_2} \qquad \ldots\ldots\ldots\ (17)$$

which is, as in all passive linear networks, independent of R_S. For iterative operation, $R_{12}=R_R$. Inserting this condition in eqn. (17) gives

$$R_1\ R_2=R_R^2 \qquad \ldots\ldots\ldots\ (18)$$

as the condition for iterative design. It is then readily shown from eqn. (15) that the iterative insertion loss ratio is

$$A_L=\frac{R_R+R_1}{R_R-R_1} \qquad \ldots\ldots\ldots\ (19)$$

from which R_S has dropped out, as expected.

(iii) Nodal analysis is now explained. Starting with Kirchhoff's node law, the equilibrium equations are written down for each node. Then, using the branch conductances, the equations are re-written in terms of the nodal potentials. We thus have four equations. But as we are concerned only with potential *differences*, one node is taken as a reference node of zero potential, and three of the equations are used to solve for the three p.d.'s. It is then readily shown that the results are identical with those obtained by the mesh method.

(iv) The formal network equations for mesh and nodal analysis are now set out and explained*. Their general form is inferred from the equations derived for the lattice network before being written down. Self- and mutual-impedances and admittances are explained.

If the number of equations in the mesh set is n (the number of independent meshes), the number of equations in the nodal set is N (one less than the number of nodes), and the circuit has B branches, then

$$n=B-N \qquad \ldots\ldots\ldots\ (20)$$

*See, e.g., Bode, H. W., "*Network Analysis and Feedback Amplifier Design*", Van Nostrand, 1945.

Bode claims that the number of nodal equations is, in general, smaller than or equal to the number of mesh equations, but is rarely larger. Moreover, the nodes can always be seen by inspection, whereas the independent meshes are not always easily recognized. It is therefore concluded that the nodal method is the better choice in practical problems.

(v) Finally in this section of the course, the students are asked to work out other ways of solving the lattice network. Ways suggested are :—

(A) by using Π to T conversion. It is not possible to find an equivalent network this way, but one mesh R_1, R_2, R_R may be converted to a T and the output voltage thus calculated. This is not usually a shorter process than the formal methods.

(B) by finding equivalent T– or Π– networks using open- and closed-circuit resistances. This is a very short process. Students usually fail to observe that conversion is only possible if $R_2 > R_1$, and will usually quote the shunt arm of the equivalent T as $\frac{1}{2}(R_1 - R_2)$. They should be shown that the open- and closed-circuit method cannot itself detect phase-reversal, and that separate inspection should be made for this.

(C) by using the results of (B) to show the rules for taking out a common series or common shunt resistance from the lattice arms, and then removing the lattice altogether by taking out *all* the resistance of one pair of arms. This is the quickest method of all.

3. TWO-TERMINAL IMPEDANCE NETWORKS (11 lectures)

3.1. INTRODUCTORY A.C. THEORY

(i) The first four lectures are devoted to a conventional treatment of a.c. theory, the j operator, and simple impedance calculations of a purely numerical kind. The formal expressions for impedance nd admittance of series and parallel combinations of RC, RL and RLC-type are then worked out and tabulated. Series and shunt connexions are not, at this stage, mixed in one circuit ; the only parallel tuned circuit combination, for example, is that of Fig. 1(b).

(ii) Q-factor is introduced as a quality-factor, and it is explained that this conception is extended to cover the ratio of reactance to resistance in any series RC or RL circuit, or the ratio of resistance to reactance in any parallel RC or RL circuit. It is clear that Q is a function of frequency. For RLC circuits, Q-factor has to be defined with care, and it is advisable to use only the term Q_0, defined as

$Q_0 = \omega_0 L/R_{se}$ or $1/\omega_0 C R_{se}$ for series circuits .... (21)

or $Q_0 = R_{sh}/\omega_0 L$ or $\omega_0 C R_{sh}$ for parallel circuits .... (22)

where R_{se} is the series resistance and R_{sh} is the shunt resistance and $\omega_0 = 1/\sqrt{LC}$.

3.2. FREQUENCY RESPONSES

(i) For these, we reduce the complexity of the expressions by introducing linear frequency normalization with the variable x :

For RC circuits, $x = \omega CR$and $x = 1$ when $\omega = 1/CR$ .... (23)

For RL circuits, $x = \omega L/R$ and $x = 1$ when $\omega = R/L$ .... (24)

For RLC circuits, $x = \omega\sqrt{LC}$ and $x = 1$ when $\omega = 1/\sqrt{LC}$ ..... (25)

The responses so obtained are tabulated and illustrated graphically. Other methods of normalization are discussed.

(ii) Voltage and current responses are calculated for resonant circuits in series with a signal source of finite impedance. This is very straightforward, and warrants no further discussion here.

(iii) A very important group of problems is illustrated in Fig. 8.

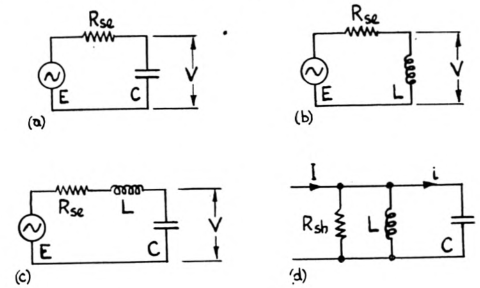


FIG. 8 E and I are given
V and i to be calculated.

These are discussed, and circuit (c) is analysed fully. We find for this circuit :

$$V = E \Big/ \left[-(x^2 - 1) + \frac{jx}{Q_0} \right] \quad \text{..........} \quad (26)$$

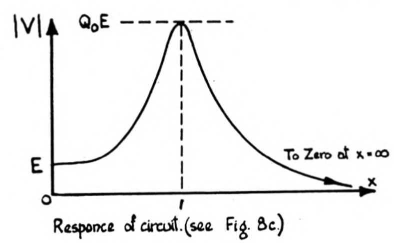
which gives an amplitude response as shown in Fig. 9. The use of Q_0 as a "magnification factor" is easily explained from this.

For the first time, the idea of the peak of the curve not occurring exactly at $x = 1$ is introduced by this circuit. Thus

$$|V| = E/\sqrt{(x^2-1)^2 + x^2/Q_0{}^2} \quad \text{..........} \quad (27)$$

and it is easily shown by differentiation that for maximum $|V|$,

$$x = \sqrt{1 - (1/2\,Q_0{}^2)} \quad \text{..........} \quad (28)$$



Responce of circuit.(see Fig. 8c)

FIG. 9.

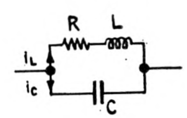


FIG. 10

(iv) It is now fitting to introduce the confusing circuit of Fig. 10. To bring the matter forcibly into the students' understanding, they are set the following exercise :

"Three possible definitions of resonance might be

(a) the condition when the impedance has its minimum or maximum magnitude,

(b) the condition when the impedance is purely resistive,

(c) the condition when the maximum energy of the magnetic field is equal to the maximum energy of the electric field. Discuss these and state which you think is the correct definition. Illustrate your answer by an analysis of the circuit of Fig. 10".

Most students answer this well. It is easily shown that for the circuit given :

Z is a maximum when $x^2 \simeq 1-(1/2\,Q_0^4)$ .... (29)

Z is purely resistive when $x^2=1-(1/Q_0^2)$ .... (30)

Max. magnetic and max. electric energy equal when $\left.\right\} x^2=1-(1/Q_0^2)$ .... (31)

3.3. MULTIPLE RESONANCE

(i) The phenomena of multiple resonance are illustrated by making reactance/frequency plots for circuits of gradually increasing complexity. The reactances or susceptances (according to the circuit arrangement) of the individual parts of the circuit are added graphically, so that the whole effect is made very clear.

(ii) The whole subject is then summarized by stating Foster's

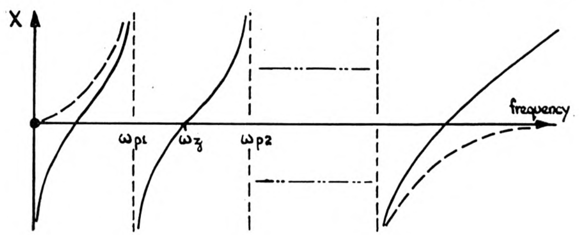


FIG 11 ILLUSTRATING FOSTER'S REACTANCE THEOREM

Reactance Theorem, illustrating it with the curves of Fig. 11, and giving (and explaining) the usual equations as follows :

$$X = \pm K \frac{(\omega^2-\omega_{z1}^2)\;(\omega^2-\omega_{z2}^2)\ldots.}{(\omega^2-\omega_{p1}^2)\,(\omega^2-\omega_{p2}^2)\ldots.} \qquad \ldots\ldots\ldots\ldots (32)$$

where $K=A/\omega$ for a pole at $\omega=0$,
$K=A\omega$ for a zero at $\omega=0$,
the $+$ sign is taken for a pole at $\omega=\infty$,
and the $-$sign is taken for a zero at $\omega=\infty$.
A is merely a numerical constant.

(iii) It now becomes clear that the whole performance of a reactance network can be expressed by the frequencies of poles and

zeros, except for the scale factor A. It is also clear than any realizable reactance/frequency relationship, however complicated, can always be realized by a network of one of the two types shown in Fig. 12(a).

(iv) The effect of adding resistance to these circuits is discussed, but only in regard to very simple arrangements, e.g. resistance connected across the terminals of the whole circuit only—none within the network.

(v) Redundancy in networks is explained and illustrated, and the ways in which series and shunt resonant circuits may be associated non-redundantly are described ; see, for example, Fig. 12(b) and Fig. 12(c). A non-redundant network has only one more element than the number of finite critical frequencies (i.e. poles and zeros) ; a network with redundancy has two or more elements in excess of the number of finite critical frequencies.

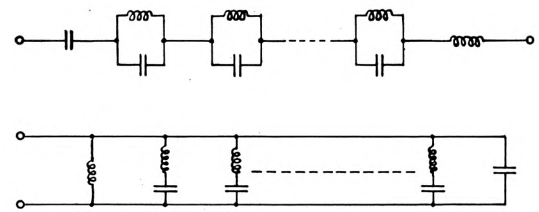


FIG. 12 a BASIC REACTANCE NETWORKS

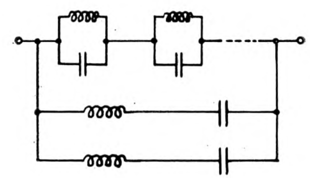


FIG. 12 b NON-REDUNDANT COMBINATION.

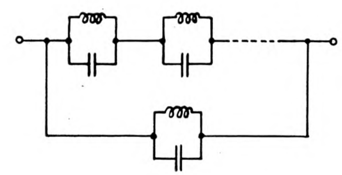


FIG. 12 c COMBINATION WITH REDUNDANCY

4. FOUR-TERMINAL IMPEDANCE NETWORKS. (10 lectures)

In this part of the course we apply the ideas of the two previous sections to the study of four-terminal networks with reactance as well as resistance in their circuits. An important group of practical networks, of great importance in communications engineering, is that where an iterative impedance which is a constant resistance at all frequencies is obtained. Such networks are used particularly as equalizers. They form a very convenient group for our initial study of four-terminal impedance networks owing to their simplicity and elegance. The ideas of image networks, which although probably more important are certainly far less elegant than the constant-resistant networks, are introduced later.

4.1. INVERSE IMPEDANCES

(i) As these form the basis of constant-resistance networks, they must be fully understood at the beginning. Fortunately the students appear to find no difficulty.

We start with some impedance Z_1 and determine the structure of an impedance Z_2 which is inverse to Z_1. To allow some flexibility later we do not say $Z_2=1/Z_1$ but define the inverse relationship as

$$Z_2=R^2/Z_1 \qquad \text{.......... (33)}$$

where R is a constant resistance.

It is easily shown that, in ordinary circuits, the rule for the construction of inverse impedances is as follows: "To form the inverse Z_2 of any network Z_1 on the basis $Z_2=R^2/Z_1$, every resistance R_x in series is replaced by a resistance R^2/R_x in shunt; every inductance L_x in series is replaced by a capacitance L_x/R^2 in shunt; and every capacitance C_x in series is replaced by an inductance C_xR^2 in shunt; and vice versa".

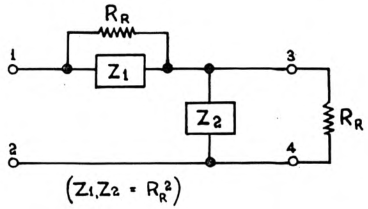


FIG. 13 CONSTANT-RESISTANCE ITERATIVE CIRCUIT

4.2. CONSTANT-RESISTANCE FOUR-TERMINAL NETWORKS

(i) The advantages of a system of four-terminal networks in which all networks have an iterative impedance which is a constant pure resistance at all frequencies is explained. It is easy to see that inverse networks have an application to this matter, since where Z_1 has a rising frequency response, Z_2 will have a falling response, and vice versa, so that the two in series would tend to be constant. But, referring to Fig. 13, the terminating impedance, which has to be a pure resistance, say R_R, appears across Z_2. It is easy to demonstrate graphically, by making Z_1 an inductance and Z_2 a capacitance, for example, that the likely solution is to connect another resistance R_R across Z_1. A formal proof that the input impedance $Z_{12}=R_R$ is then given. It is a simple further step to show that the insertion loss ratio is

$$A_L=1+\frac{Z_1}{R_R} \qquad \text{.......... (34)}$$

The insertion loss in db $=20\log_{10}\left|1+\frac{Z_1}{R_R}\right|$ .... (35)

The insertion phase-shift $=\tan^{-1}[X_1/(R_R+R_1)]$ .... (36)
where $Z_1=R_1+jX_1$.

(ii) The history of this type of network, from its publication by Zobel in 1928, is briefly given.

(iii) The development of constant-resistance networks into other configurations, such as T, ∏, Bridged-T and lattice, is dealt with in as much detail as time permits; and a summary sheet, based on Fig. 111 of Terman*, is issued. It is explained that, of all these forms, only the ˥, Bridged-T and lattice are of practical importance; and as far as the lattice is concerned, only the final type shown—without explicit resistances—is important. The special features of this final network are discussed in some detail. These include

(a) A constant-resistance input is still obtained even if Z_1 and Z_2 are pure reactances. Since pure reactances cannot dissipate energy, then $|A_L|=1$ and only phase-shift, not loss, is produced by the network.

(b) The equation for A_L is different for this network; it is

$$A_L=\frac{R_R+Z_1}{R_R-Z_1} \qquad \text{.......... (37)}$$

This expression shows that it is possible for this network to give A_L a negative value—i.e. it can give a phase reversal.

(iv) The application of constant-resistance networks to equalization of cable attenuation, etc., is discussed, and methods of design are outlined. The author uses here the material contained in his old paper, "*Constant-Impedance Networks for Line Equalization*", *P.O. Elect. Engrs. J.*, **29**, Jan. 1937, p. 302. Although this paper is now largely out-of-date as far as design specialists are concerned, it is still suitable for students who want only the simple ideas. A numerical example is worked through, to illustrate the sort of "cut-and-try" design methods which engineers so often have to adopt; and the advantages of having families of normalized curves already available are pointed out.

(v) The application of the lattice network as an "all-pass" phase equalizer is examined. If $Z_1=jX_1$ then

$$A_L=\frac{R_R+jX_1}{R_R-jX_1} \qquad \text{.......... (38)}$$

from which we obtain

$$\tan\frac{\theta}{2}=\frac{X_1}{R_R} \qquad \text{.......... (39)}$$

where θ is the phase-shift given by the network. Graphs of phase-shift against frequency are plotted out for simple types of Z_1, and their application to phase equalization of relatively small phase errors is described.

The restrictions placed on the phase-shift/frequency response by the fact that $\tan\frac{\theta}{2}$ has to follow Foster's Reactance Theorem [because of eqn. (39)] are pointed out.

*Terman, F. E., "*Radio Engineers' Handbook*", McGraw-Hill, 1943.

The use of Foster's Reactance Theorem in the synthesis of a phase equalizer for large phase errors is discussed. The ideas of synthesis, as opposed to analysis and cut-and-try design, are emphasized. A simple case to take is that of a phase equalizer for a cable, as discussed by Guillemin*.

4.3. FILTERS : THE NEED FOR IMAGE NETWORKS

(i) If understanding of networks is to be achieved, it is no good merely telling the student that filters are designed on an image basis and equalizers on an iterative basis. He needs to know why. Text-books appear to ignore this important question.

It is suggested that the main requirements of filters are :

(a) A circuit must be branched into two or more separate circuits, each of which passes a different band of frequencies, with no wastage of energy and yet with efficient separation,

(b) For each filter there should be a distinct frequency *band* over which acceptance is substantially complete, and as near a discontinuity as possible between this band (the "pass" band) and the band over which rejection is required,

(c) It must be possible to cascade filter networks so that an increased rejection ratio is obtained.

Now it is clear that a constant-resistance iterative network cannot meet requirement (a), since it absorbs energy into its input resistance at all frequencies. Thus if there are 12 branch circuits, each receives only $^{1}/_{12}$th of the available power. It is clearly a requirement of filters that power should be absorbed into the input terminals only at those frequencies where it is to be passed on to the load, and this means that the input impedance must be predominantly resistive in the pass band and a pure reactance elsewhere. If the various filters are to be connected in parallel, then the reactance should obviously be high ; if they are to be connected in series, then it should be low.

As regards requirement (b), it is not possible to obtain a discontinuity in the insertion loss/frequency response of an iterative network of the constant-resistance type. This is clear from eqn. (34), since Z_1 cannot be made to be zero over a band, and then to change suddenly to a finite value.

We must now conclude that a different basis of network operation is needed if ladder filters are to be realized.

(ii) The basis of filter design invented by Zobel (and first published in 1922) is the *image* network. Historically, this preceded the iterative network—which is rather curious. Image networks are cascaded on the basis that the impedances looking both ways at any junction are the same. This is indicated in Fig. 14. The impedances at different junctions may be different. The impedances Z_0 and Z_0' are called image impedances. A convenient, if inelegant, definition of image impedance is "the input impedance of a network when its output is terminated by the image impedance of the output."

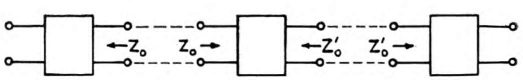


FIG. 14. IMAGE NETWORKS IN CASCADE

*Guillemin, E. A., "*Communication Networks*", Vol. 2, Chap. XII, p. 538, Wiley, 1935.

(iii) Instead of insertion loss, the transmission properties of the image network are measured by the image propagation constant (P), image attenuation (A), and image phase-shift (B), where

$$e^{P}=e^{A+jB}=\sqrt{\frac{V_{in}\,I_{in}}{V_{out}\,I_{out}}} \qquad \dots\dots\dots\dots (40)$$

The "root-volt-amp" ratio is used because input and output impedances are neither constant nor identical.

(iv) It is now easy to show that if only reactances are included in the network, then A can be zero over a finite frequency band, and have a finite value outside that band with a discontinuity at the edge of the band. It is also easy to show that the image impedance Z_0 is discontinuous at the edge of the band, being a pure resistance within the band, and a pure reactance outside it. This is standard text-book work ; the pass band occurs when the open- and short-circuit impedances at the input terminals are reactances of opposite sign, and the stop band occurs when they have the same sign.

The stop and pass band behaviour of various simple filter networks can now be easily obtained by making reactance/frequency plots of open- and short-circuit impedances, and noting where the signs change.

5. CONCLUSIONS

It will be seen that the course outlined here is not at all conventional in its main features, yet it covers most of the ground traditionally covered in examination courses in electrical networks. Although only an introductory course it has dealt with all common network configurations, iterative and image operation, insertion loss and image attenuation, equalization and filtration, analysis and synthesis and design by "cut-and-try"; it has presented the usual formal network theorems and the usual formal network equations including the less usual nodal equations ; it has introduced and made use of Foster's Reactance Theorem. Above all, it seems to have been accepted readily by students.

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