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The Early History of Amplitude Modulation, Sidebands and Frequency-Division-Multiplex

By

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It is shown that the ideas of f.d.m. originated with Alexander Graham Bell around 1870 and were formulated as an f.d.m. telephone system by Leblanc in 1886. Amplitude modulation of a carrier by speech probably originated also with Leblanc in 1886. The existence of sidebands (or sidetones) was demonstrated experimentally by Mayer in 1875 and theoretically by Rayleigh in 1894, but was not known to the early radio and telephone engineers, being apparently rediscovered in 1915. The main developments up to about 1920 are briefly discussed.

1. Introduction

The early history of carrier telephony as such is, on the whole, reasonably clearly presented in the literature; and in particular the long paper by Colpitts and Blackwell[1] of 1921, although it is misleading in one or two respects concerning the earlier history, includes an excellent historical survey from about 1890 onwards. There is little need, therefore, to repeat this part of our history in anything but outline form. When, however, we look at the way the basic ideas developed—that is, the ideas of frequency-division-multiplex, of amplitude modulation, and of sidebands—we find no very clear picture. Indeed, the story of sidebands is most remarkable. Writers on the history of modulation, such as Heising,[2] state that the ideas of sidebands were developed around 1915; certainly radio and electrical workers before that time appeared completely ignorant of sidebands. Yet sidebands were experimentally demonstrated by Mayer[3] in 1875 and theoretically and experimentally demonstrated by Rayleigh[4] in 1894, in both cases in the context of acoustics.

In this paper we set out to describe the early history of the subject in reasonable perspective, and the description will be aided by the diagrammatic summary of Fig. 1.

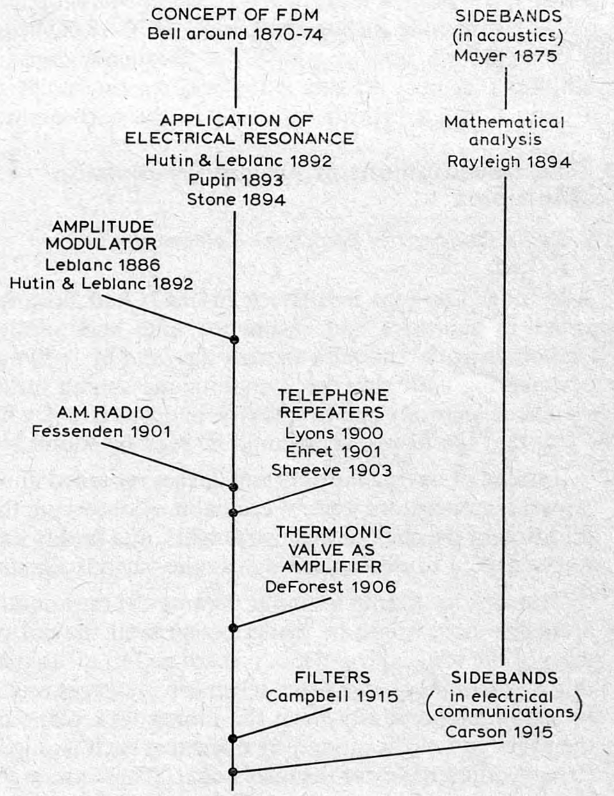

Fig. 1. Diagrammatic summary.

2. Early Ideas of Multiplexing in Telegraphy and their Influence on Telephony

Before starting on our main subject, it is interesting to note that several methods of multiplexing telegraph channels were proposed and/or developed, and three of these proved to be the forerunners of important telephone multiplexing methods. 'Duplex' telegraphy was the name given to a system permitting messages to be sent simultaneously both ways over a single line; it was invented in 1853 by Gintl, but there were many other similar inventions around the same time, including one by Lord Kelvin using capacitors to enable duplex working to be achieved on cables. A good account of this piece of history is given by Bright.[5] The essential principle was a differential coil, or later a balanced bridge, to prevent a transmitted signal from disturbing the receiver at the transmitting station. The same principle was later used in amplified telephone lines, using what became known as the 'hybrid' coil to separate 'go' and 'return' channels at the terminals and at intermediate repeater stations.[6]

'Quadruplex' telegraphy,[7] developed by Edison in 1874, although first proposed in 1855, was a method of transmitting two messages each way over a single line, and was achieved by adding to the duplex system a method of distinguishing two simultaneous messages by what amounted to putting a d.c. bias on one of them. There was no telephone application of this.

'Multiplex' telegraphy,[7] successfully developed by Baudot in 1875, used a mechanically-driven rotary switch to sample a number of messages successively at an appropriate speed, and a synchronous switch at the receiving end distributed the samples to the correct receiving channels. This was what is now called 'time-division multiplex' (t.d.m.). After some misunderstanding of its possible application to telephony by people such as Leblanc[8] who failed to appreciate the need for a very high speed of sampling, it was Miner[9, 10] who in 1903 showed that it could work on speech signals if the

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sampling frequency was made higher than the frequencies contained in the speech. In his own experiments he used a sampling frequency of 4320 Hz. He did not fully understand the sampling requirement, for his patent claim No. 1 was:

> 'The herein-described improved art of multiplex telephony consisting in synchronously closing the connection between the line and corresponding branches or subcircuits with a frequency corresponding to the frequency of the tones and overtones characterizing speech.'

According to Black,[11] it was left to Carson in 1920 to develop the true sampling theorem on which modern t.d.m. systems are based.

Finally, a system of multiplex telegraphy was proposed by Bell[12] sometime during the period 1870–1874, using the principle we now recognize as 'frequency-division multiplex' (f.d.m.). As this is the real starting-point of our main study, it is dealt with fully in the next section.

3. The Development of Frequency-division Multiplex

3.1 *Bell's Concept of Multiplex Telegraphy using F.D.M.*

Alexander Graham Bell (born in 1847) had been interested in acoustics and resonance and had studied Helmholtz's work[13] at quite an early age, and by 1870 was experimenting with electrically-maintained tuning forks. He studied electricity and telegraphy, and was struck with the fact that the Morse code could be read by sound.[12]

> 'Instead of having the dots and dashes recorded upon paper, the operators were in the habit of observing the duration of the click of the instruments, and in this way were enabled to distinguish by ear the various signals.
>
> 'It struck me that in a similar manner the duration of a musical note might be made to represent the dot or dash of the telegraph code. . . . It seemed to me that in this way a number of distinct telegraph messages might be sent simultaneously from the tuning fork piano to the other end of the circuit, by operators each manipulating a different key of the instrument. These messages would be read by operators stationed at the distant piano, each receiving operator listening for signals of a definite pitch, and ignoring all others. In this way could be accomplished the simultaneous transmission of a number of telegraphic messages along a single wire, the number being limited only by the delicacy of the listener's ear. . . .'

This is clearly a proposal for an f.d.m. system. Of course, Bell had no idea of the factors limiting the number of channels he could obtain. He refers to the 'delicacy of the listener's ear' as the limitation. Later on in the paper cited[12] (p. 400) he says, referring to the use of undulatory currents (i.e. sine waves) as what we would now call carriers:

> 'Hence it should be possible to transmit as many musical tones simultaneously through a telegraph wire as through the air.'

He goes on to describe the 'electric harp' as an application of the principle and describes it as 'my first form of articulating telephone'; so we might say that his first

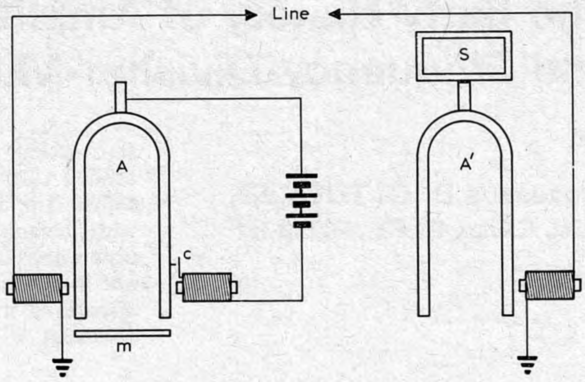

Fig. 2. Leblanc's amplitude-modulation system of 1886.
A transmitting tuning fork
A′ receiving tuning fork
S sounding box
c make-and-break contact in maintaining circuit
m modulating plate or diaphragm

telephone proposal was derived from f.d.m. telegraphy. He also proposed the application of f.d.m. to provide one axis of scanning for picture telegraphy (see p. 395 of paper cited[12]).

The application of these ideas of f.d.m. to telephony appears to have originated with Leblanc,[8] who in his paper of 1886 describes very clearly a system of amplitude-modulation and its application to f.d.m. telephony. Figure 2 is redrawn from his paper. An electrically-maintained tuning fork (A) generates the supersonic carrier-current wave, and the amplitude of this in the line is modulated by varying the mean (or biasing) magnetic field at the pick-up coil by vibrating an iron plate (m) in the vicinity of the tines. This iron plate can be made the diaphragm of a microphone. Leblanc actually draws (in another figure) the waveform of the amplitude-modulated signal. Reception is effected by a tuning fork (A′), not self-maintained but driven by a coil carrying the line current, and connected to a sounding box (S). By using several such arrangements, each with a different frequency, multiplex telephony can be obtained.

Leblanc was evidently aware of the difficulty of getting a highly-resonant system such as a tuning fork to respond fast enough to reproduce the envelope of a speech-modulated wave, for he says the fork A′ must be very light:

> 'Le diapason A doit être très lourd, et le diapason A′ au contraire très léger.
>
> 'En effet, il faut que l'inertie du diapason A soit assez grande pour qu'une fois ébranlé, il continue à vibrer pendant la durée d'une conversation, d'un autre côté le diapason A′ doit avoir aussi peu d'inertie que possible pour que l'amplitude de ses vibrations soit à chaque instant proportionelle à l'intensité des courants phoniques de pèriode convenable qui parcourent la ligne.'†

† 'The tuning-fork A should be very heavy, and A′ on the contrary very light.
'Actually the inertia of fork A must be great enough to keep it vibrating, once started, for the duration of a conversation; on the other hand fork A′ must have as little inertia as possible so that the amplitude of its vibrations may be proportional at each instant to the intensity of the phonic currents of suitable period which traverse the line.'

The frequency of the fork vibration was to be chosen to be above the audible range, which Leblanc understood to extend up to 8 kHz. There is no sign, however, that he was aware of the idea of sidebands.

The f.d.m. systems of Bell and Leblanc clearly depended on mechanical resonance for separating the channels. The use of electrical resonance for this purpose, discovered around 1892–4, was a big step forward. There seem to have been several almost simultaneous proposals, by Hutin and Leblanc[14] in France, and by Pupin[15, 16] and by Stone[17] in the U.S.A. The idea of the complete f.d.m. telephone system with electrical tuning appeared in Hutin and Leblanc's proposals. However, the fact that they thought a 2 kHz spacing of channel centre frequencies to be suitable suggests that they made no practical trials of their system.

3.2 *The Beginning of Amplitude Modulation in Telephony*

Clearly the application of f.d.m. to telephony depended on the development of ideas of amplitude modulation of a carrier wave and the corresponding process of detection. We have mentioned Leblanc's tuning-fork method above. The first such development using electrical methods appears to be by Hutin and Leblanc[14] in 1892. Their system is shown in Fig. 3. Here G is the h.f. generator (a commutator device), T is the microphone used as a modulating element, modulating the h.f. current in the same way as it would normally modulate a direct current in an ordinary telephone, and DR is a dynamometer receiver which by virtue of its square-law action virtually rectifies the a.m. wave and produces an acoustic output derived from the envelope. This appears to be in all essentials a workable a.m. system.

The application of amplitude modulation to radio seems to have been an independent but later development and here Fessenden[18] seems to have been the pioneer. He proposed (in 1901) basically two ways of obtaining a.m. One was the use of a magnetic or dielectric modulator to detune the aerial by an amount dependent on the speech current and thus reduce (i.e. modulate) the proportion of the generator current which flowed in the aerial. The other, which seems to have been used in all his effective radio-telephone experiments, was to use the resistance of a carbon microphone to modulate the aerial current directly in more-or-less the same way as used by Hutin and Leblanc for line telephony.

The idea of using a rectifier of some sort as a detector for a.m. signals appears to have been the invention of Pupin[19] in 1898. The rectifier was an electrolytic cell, and it was telegraph rather than telephone signals that Pupin was concerned with.

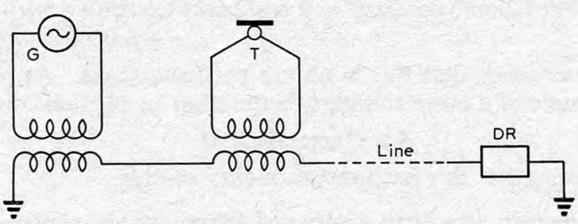

Fig. 3. Hutin and Leblanc's system of amplitude-modulation and detection

G h.f. generator T microphone DR dynamometer receiver

There seems to have been some interaction of radio and line telephony in the period following Fessenden's early work, although it is clear that the correct attribution of ideas was not always made. It is interesting, for example, that Ruhmer[20] said in 1907:

> 'At the same time it is possible that wireless telephony will have a considerable influence on the development of wire-telephony. We may notice, for instance, the problem of multiplex telephony, the solution of which may lie in the adoption of wireless methods.'

Ruhmer himself took out a Belgian patent[21] in 1910 of which the official abridgement is:

> 'On emploie des courants alternatifs à haute frèquence et de frèquence différente, qui se superposent sur la ligne et ne sont séparés qu' á la station réceptrice en agissant là sur différents circuits oscillatoires accordés avec les circuits oscillatoires de transmission correspondants.'†

This seems hardly an advance over the concepts of 1892. But his experiments show a big advance in detailed technique through the adoption of radio methods.[22]

Ruhmer also discusses in his book[23] the possibility of using f.d.m. telephony over a light-beam link, using a modulated arc-light for transmitting and a selenium cell as detector, with light filters for the various channels.

An important discussion of f.d.m. on lines appeared in 1911 in the paper by Squier,[24] who also held some patents on f.d.m.[25] He was concerned with working on cable, superposing carrier channels on the existing 'low frequency battery system', i.e. on the existing audio channel. He used tuned circuits for filtering, and gave many experimental measurements of the response of tuned lines.

The techniques of modulation changed considerably when the thermionic valve became available. The early triode valve, or 'audion', could be used for modulating a carrier wave, and a good account of the development of modulation using valves is given by Heising.[26] Similarly the diode valve could be used as a detector, although for some time after its invention the crystal detector was preferred as being more reliable and much cheaper; it was discovered by Dunwoody[27] in 1906 and then based on carborundum (silicon carbide). Pickard[28] followed this up rapidly with a choice of other materials, and many more were subsequently used as crystal detectors.[29, 30] The modern semiconductor rectifiers and transistors are directly derived from the crystal detector.

3.3 *The Influence of Telephone Repeaters*

The influence of telephone repeaters (or amplifiers) on the development of f.d.m. telephony was for a long time negligible. A good history of the telephone repeater was given by Gherardi and Jewett[6] in their paper of 1919.

† 'High-frequency alternating currents are used, of different frequencies, which are superposed on the line and are separated only at the receiving station. There they act on different oscillatory circuits lined up with the corresponding oscillatory circuits at the transmitting station.'

They suggest that the first practical telephone repeater was a receiver/transmitter type (i.e. an electrodynamic repeater based on telephone instruments) developed by H. E. Shreeve in 1903, and they also mention that in 1900 J. Lyons disclosed to the Bell System a proposal for a repeater based on an asynchronous generator, i.e. an induction motor driven above synchronous speed by a local prime mover. Such an arrangement is, in effect, a negative-resistance amplifier. Probably neither of these repeaters would have been suitable for f.d.m. telephony on account of frequency range. It is interesting though that Ehret,[31] who also invented frequency-modulation,[32] proposed in 1901 an f.d.m. telephone system involving line repeaters of the asynchronous-generator type.

The invention of the triode thermionic valve by De Forest[33] in 1906, following earlier work by others on the diode, opened the door to more effective and useful telephone repeaters. The trouble was that, although the frequency range could now be obtained for f.d.m. working, non-linearity in the response prevented satisfactory working of multiple channels because of the intermodulation-crosstalk produced. Early carrier (or f.d.m.) telephone systems operated on open-wire lines, without intermediate amplification. Some success was obtained with two-channel systems with intermediate repeaters in which the non-linearity was nominally cancelled by the insertion of a complementary non-linearity produced by rectifiers.[34] However, it was not until Black made his ideas on the negative-feedback amplifier[35] generally available in 1934 that multi-channel systems could operate really successfully over amplified lines, and it was from that date that the really important commercial development of f.d.m. telephony began.

4. The Curious History of Sidebands

4.1 *The Radio and Line Telephony Era*

As stated briefly in the Introduction, the story of the discovery of sidebands (or sidetones) is very curious. Radio workers were apparently quite unaware of the concept of sidebands and of the bandwidth requirements of a modulated signal for practically two decades from the first successful radio experiments of Marconi and others. Heising,[2] who was himself a pioneer of radio technology, said:

> 'No equation for a modulated wave is found in the literature prior to World War I. So far as the author has any direct knowledge, the equation was first set up and side frequencies discovered by Carl Englund in 1914. The disclosure to the armed forces during World War I and to the public afterward stemmed from Englund's discovery. It is true that some engineers of the American Telephone and Telegraph Company seem to have had earlier knowledge of the sidebands from their study of carrier-system theory for wire lines, but the perusal of their memoranda and other Company writings has not turned up a prior discoverer. Dr. G. A. Campbell (A. T. & T. Company) believed their knowledge was derived from mathematical expressions in Rayleigh's "Theory of Sound" involving periodically interrupted sound tones.'

If Heising is right in stating that Englund was the first to discover sidebands in the radio context, then Englund was slow to patent a use for his discovery. The first disclosure by Englund that the author can trace is a U.S. Patent[36] filed on 29th March 1916, which mentions that a carrier wave of frequency C modulated by a signal of frequency S gives rise to three frequencies $C-S$, C, and $C+S$, and mentions carrier suppression and re-insertion at the receiver. But in the meantime, Carson had filed a patent[37] on 1st December 1915 which gives a full mathematical theory of the production of sidebands of various types in a non-linear device (i.e. a modulator) and also shows the benefit, not only of carrier-suppression but also of single-sideband operation. It looks very much to the present author as though Carson was the real discoverer (or re-discoverer) of sidebands in radio. Carson also later published a paper[38] on this work. It was Carson who also first demonstrated the sideband structure of frequency-modulation.[39]

An interesting demonstration of how little understanding of sidebands there was in the period prior to 1920 is a mistaken idea of them in the book by the very reputable radio engineer Goldsmith.[40] He says:

> 'If a 100,000 cycle sustained wave be modulated by a 1000-cycle note, both theory and practice agree as to the propriety of regarding the modulated wave as the resultant of *three* separate waves: namely, one corresponding to the frequency of 100,500, one corresponding to the frequency of 99,500, and one corresponding to the frequency of 100,000. All three, being physically present, are detectable with a wave meter, and this has a certain bearing on the selectivity in radio telephony, particularly at very long wave lengths, corresponding to low radio frequencies.'

4.2 *The Real Discovery of Sidebands, 1875 onwards*

It is probable that the first reference to sidebands (although not by that name) is in the paper by Mayer[3] published in 1875. He describes experiments in acoustics in which he modulates or interrupts the sound from a tuning fork by means of a rotating screen with holes in it. As the speed of rotation is increased from zero he notices two additional sounds appear:

> 'On revolving the perforated disk, two additional or secondary sounds appear—one slightly above, the other slightly below the pitch of the fork.'

This is a very clear picture of sidetones.

Lord Rayleigh did some further work on this subject, and set out the theory clearly in the second edition of Vol. 1 of his famous book[4] in 1894. He gives in effect the equation

$$2(1+\cos 2\pi mt)\cos 2\pi nt = 2\cos 2\pi nt+\cos 2\pi(n+m)t+ +\cos 2\pi(n-m)t$$

and explains that this is only a particular case. As an example of a more complex modulation he expands

$$4\cos^4 \pi mt \cos 2\pi nt$$

showing that this has four secondary sounds.

Rayleigh also gives a very satisfying physical explanation of the secondary sounds. He describes first of all an experiment in which a tuning fork of frequency 128 is driven by a current which is interrupted at frequency 128 by a fork-driven interrupter. This current can also be interrupted by another independent interrupter of frequency 4. When the second interrupter was inoperative, the fork had a strong response in its normal tuning of 128, but scarcely any when tuned to 124 or 132. When the second interrupter was working, however, the fork would respond powerfully when tuned to 124 or 132 as well as when tuned to 128, but not when tuned to intermediate pitches, such as 126 or 130.

The physical explanation which Rayleigh then gives is:

'When a fork of frequency 124 starts from rest under the influence of a force of frequency 128, the impulses cooperate at first, but after 1/8 of a second the new impulses begin to oppose the earlier ones. After 1/4 of a second another series of impulses begins whose effect agrees with that of the first, and so on. Thus if all these are allowed to act, the resultant effect is trifling; but if every alternate series is stopped off, a large vibration accumulates.'

This is a very helpful way of looking at sidetones.

It is thus more than likely that Campbell, quoted by Heising, was right in believing that the A. T. & T. Company's knowledge of sidebands was derived from Rayleigh. What is surprising is that so few radio workers were aware of Rayleigh's work.

4.3 *Filters*

The re-discovery of sidebands led to proposals for single-sideband working and to the desire for better methods of selecting frequency bands than the simple tuned-circuit arrangements used up till then. Thus gradually the art and science of filter design developed. Probably the first filter invention was that by Campbell[41] in 1915. The invention of m-derived filters by Zobel[42] in 1923 was a most important step, and led to very refined filter design.

5. References

1. Colpitts, E. H. and Blackwell, O. B., 'Carrier current telephony and telegraphy', *Trans. Amer. Inst. Elect. Engrs*, **40**, pp. 205–300, 1921.
2. Heising, R. A., 'Modulation methods', *Proc. Inst. Radio Engrs*, **50**, pp. 896–901, May 1962.
3. Mayer, A. M., 'Researches in acoustics, Part 6', *Phil. Mag.*, **49**, pp. 352–365, May 1875.
4. Rayleigh, Lord, 'The Theory of Sound', Vol. 1, Second edition (Macmillan, London, 1894).
5. Bright, C., 'Submarine Telegraphs', see p. 122 and pp. 635–43 (Crosby, Lockwood & Son, London, 1898).
6. Gherardi, B. and Jewett, F. B., 'Telephone repeaters', *Trans. Amer. Inst. Elect. Engrs*, **38**, pp. 1287–1345, 1919.
7. Denman, R. P. G., 'Line Telegraphy & Telephony', Catalogue of the Collections in the Science Museum, South Kensington (H.M.S.O., 1926).
8. Leblanc, M., 'Etude sur le téléphone multiplex', *La Lumière Électrique (Paris)*, **20**, pp. 97–102, 17th April 1886.
9. Miner, W. M., U.S. Patent No. 745,734, filed 26th February 1903, issued 1st December 1903.
10. 'Recent developments in multiplex telephony,' *Electrical World & Engineer (U.S.A.)*, **42**, p. 920, 5th December 1903.
11. Black, H. S., 'Modulation Theory', see p. 5 (Van Nostrand, New York, 1953).
12. Bell, A. G., 'Researches in electric telephony', *J. Soc. Telegraph Engrs*, **6**, pp. 385–421, 1877.
13. Helmholtz, H. L. F., 'Die Lehre von dem Tonempfindungen als physiologische Grundlage für die Theorie der Musik', 1863. English translation by A. J. Ellis, 'Theory of Tone'.
14. Hutin, M. and Leblanc, M., British Patent No. 23,892; Application date 27th December 1892.
15. Pupin, M. I., 'Practical aspects of low frequency electrical resonance', *Trans. Amer. Inst. Elect. Engrs*, **10**, pp. 370–94, 1893.
16. Pupin, M. I., U.S. Patent No. 707,007, filed 23rd February 1894, issued 12th August 1902.
17. Stone, J. S., U.S. Patents 726,368; 726,476; 729,103; 729,104; all filed 4th April 1894, issued 1903.
18. Fessenden, R. A., U.S. Patent No. 753,863, filed 28th September 1901, issued 8th March 1904.
19. Pupin, M. I., U.S. Patents Nos. 713,044 and 713,045, filed 4th January 1898, issued 4th November 1902.
20. Ruhmer, E., 'Wireless Telephony, in Theory and Practice', published in Germany 1907; English translation by J. Erskine-Murray, published by Crosby, Lockwood & Son, London, 1908. See p. 194.
21. Ruhmer, E., Belgian Patent No. 224,008, dated 16th March 1910.
22. Ruhmer, E., 'Multiplex telephony', *Electrical Review and Western Electrician*, **59**, pp. 28–29, 1st July 1911.
23. Ruhmer, E., as ref. 20, p. 73.
24. Squier, G. O., 'Multiplex telephony and telegraphy by means of electric waves guided by wires', *Trans. Amer. Inst. Elect. Engrs*, **30**, pp. 1617–65, 1911.
25. Squier, G. O., U.S. Patents Nos. 980,356; 980,357; 980,358; 980,359, all issued in 1911.
26. Heising, R. A., 'Modulation in radio telephony', *Proc. Inst. Radio Engrs*, **9**, pp. 305–352, 1921.
27. Dunwoody, H. H. C., British Patent No. 5332, 23rd March 1906.
28. Pickard, G. W., U.S. Patent No. 836,531, filed 30th August 1906, issued 20th November 1906 (silicon crystal); U.S. Patent No. 886,154, filed 30th September 1907, issued 28th April 1908 (zincite).
29. Pierce, G. W., 'Principles of Wireless Telegraphy', see pp. 160–1 (McGraw-Hill, New York, 1910).
30. Palmer, L. S., 'Wireless Principles and Practice', see pp. 302–308 and many references quoted on pp. 332–3 (Longmans, Green & Co., London, 1928).
31. Ehret, C. D., U.S. Patent No. 789,087, filed 3rd December 1901, issued 2nd May 1905.
32. Tucker, D. G., 'The invention of frequency-modulation in 1902', *The Radio and Electronic Engineer*, **40**, pp. 33–37, July 1970.
33. DeForest, L., 'The audion', *Trans. Amer. Inst. Elect. Engrs*, **25**, pp. 735–63, 1906.
34. Ryall, L. E., 'A Few Recent Developments in Telephone Transmission Apparatus', Inst. P.O. Elect. Engrs Printed Paper No. 155, 1934, see pp. 18–20.
35. Black, H. S., 'Stabilized feedback amplifiers', *Bell Syst. Tech. J.*, **13**, pp. 1–18, 1934, or *Electrical Engr*, **53**, pp. 114–20, 1934.
36. Englund, C. R., U.S. Patent No. 1,245,446 filed 29th March 1916, issued 6th November 1917.
37. Carson, J. R., British Patent No. 102,503, Convention date (U.S.A.) 1st December 1915, issued 30th November 1917.
38. Carson, J. R., 'A theoretical study of the three-element vacuum tube', *Proc. Inst. Radio Engrs*, **7**, pp. 187–200, 1919.
39. Carson, J. R., 'Notes on the theory of modulation', *Proc. Inst. Radio Engrs*, **10**, pp. 57–64, 1922.
40. Goldsmith, A. N., 'Radio Telephony', p. 181 (The Wireless Press, New York, 1918).
41. Campbell, G. A., U.S. Patents Nos. 1,227,113 and 1,227,114, filed 15th July 1915, issued 1917; or British Patent No. 142,115, issued 1921.
42. Zobel, O. J., 'Theory and design of uniform and composite electric wave-filters', *Bell Syst. Tech. J.*, **2**, pp. 1–46, 1923.

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